THE PRINCIPLES OF LOGIC

DEDUCTIVE AND INDUCTIVE

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To

My Mother.

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BOOK III.

INDUCTION.

Division I.

CHARACTER AND CONDITIONS OF INDUCTION.

CHAPTER XV.

CHARACTER OF INDUCTION.

§ 1. **Transition to Induction.** We have seen that in actual life Induction and Deduction run into each other—Induction proving the material validity of abstract truths established by Deduction, and Deduction also supplementing and verifying inductive generalizations and showing their harmony. In passing from facts to laws, we employ the inductive method; while, in applying these laws to new cases, we make use of the deductive method. We have seen also that, in syllogistic reasoning, at least one of the premises must be universal, which, if not a fundamental truth, must have been reached by induction. Thus, if we are not satisfied with the mere formal truth secured by deduction, we are inevitably led to inquire into the correctness of the data and thus into the inductive validity of the universal premise. If our end is truth in the full sense of the term—and not this or that form of truth—then we can never be satisfied with an examination of this or that type of reasoning, but we must inquire into the grounds of all reasoning contributing to a

Induction and Deduction supplement each other in conclusively establishing a truth.

Truth in the full sense can be attained when Deduction is aided by Induction.

complex result. Hence the transition from Deduction to Induction is as natural and necessary as the passage from Induction to Deduction. Indeed they are but different aspects of the inferential activity. And, since we have discussed Deduction first, for its comparative simplicity, we must now pass on to the consideration of Induction, which supplies universal propositions, serving as premises of deductive reasoning.

Induction inquires into the (material) truth of universal propositions entering into deductive reasoning.

Characteristics of Induction:

§ **2. Marks of Induction.** An Inductive Inference implies that we arrive at a universal proposition in harmony with facts, by observing a number of individual instances. It is a universal real proposition based on observation and established in conformity with the uniformity of Nature. Three points we should note in this connection :—

(1) Induction establishes a proposition instead of a notion.

(1) What is established by Induction is a proposition as distinguished from a notion. A notion often involves but a single idea or quality, while an inductive proposition expresses a connection between two notions or terms. Sometimes, no doubt, a notion may be complex, involving a plurality of ideas or qualities, *e.g.*, 'man', 'matter', 'book'; and in such notions the ideas or qualities are evidently found connected. But there is a difference even between such a complex notion and an Induction. In a complex notion the connection is tacitly assumed, whereas in Induction the connection is open to question and has thus to be proved. (*Vide* Chap. XXV, § 1.)

In a complex Notion, a connection of elementary ideas is assumed, while in an Induction, it has to be proved.

(2) In Induction the

(2) The conclusion in Induction is always more general than the data or premises: we observe

only a few cases, while the inference covers all similar cases. The essence of Induction lies, as indicated by Mill and Bain, in the leap or hazard involved in passing from the known to the unknown. If in any case there is no such leap, we cannot call it Induction proper. If, for example, on observing individually that every student of this class prepares his lesson, I conclude that all the students of the class prepare their lesson, it cannot be called Induction in the proper sense of the term. The conclusion here does not state more than what is given in the premises: the conclusion is but an epitome or summary of the instances observed. This form of Induction is described by Mill and Bain as Improper Induction. A true induction indicates that we pass from 'some' to 'all'—from the cases which are present before us to all similar cases, embracing the past, the present, and the future, the near and the distant, the familiar and the unfamiliar. Thus, when we conclude that '*all* material bodies have weight' from '*some*' known instances of the weight of such bodies, we argue inductively in the proper sense of the term.

conclusion is more general than the data.

(3) Induction must ultimately be based on observation. We have seen that formal truth or self-consistency is the end of Deduction, while material truth or conformity to fact is the end of Induction. A generalization not in harmony with fact is to be rejected as an extravagant hypothesis.

(3) Induction aims at material truth, and so it is based on observation.

§ **3. Postulates of Induction.** We have seen in Chapter I, § 10, that Logic presupposes

Besides the general postulates of Logic

mentioned in Chap. I, § 10, Induction assumes the coherent and systematic character of the world with the necessary implications of the Principles of Causation, Uniformity and Sufficient Reason.

certain data without which there would be neither materials nor principles to proceed with. These ultimate data are, as we have said, (1) the mind or the subject, (2) the thing known or the object and (3) the relation between the subject and the object. In addition to these assumptions there are certain others in all Inductive inferences. We have seen that Induction aims at establishing universal real proposition from the particular facts of experience. It thus tries to discover the laws of nature which express general connections between several phenomena which would otherwise seem detached. It, therefore, assumes that there are fixed and definite relations in nature, that the world is not a chaos but a cosmos where every event has a sufficient reason or cause for its happening. This assumption is expressed in the principles of Sufficient Reason, Causation and Uniformity of Nature which are regarded as the ground of Induction. We have seen that all these principles are but expressions of Consistency or Identity. (*Vide* Chap. II, §§ 7-9). Law of Uniformity, for example, as found in Nature is really Identity in another form and Law of Causation as understood in science is based on the same principle. Similarly, the Principle of Sufficient Reason aiming to show necessary and identical connection between effects and causes or conclusions and data is but Identity differently expressed. Hence, the most general principle which lies at the root of all Inductions is the Principle of Identity.

§ 4. Perfect and Imperfect Induction. A distinction has been drawn between Perfect and Imperfect Induction. An Induction is said to be *perfect* when a universal proposition is established after an examination of *all* the instances coming within its sweep. When, for example, we say that all the students of this class have prepared their lesson well after examining every one of them, then the induction is said to be perfect, because there is perfect assurance about the conclusion: when we have observed every case coming within the 'all', the universal conclusion cannot but be true. Hence, Aristotle regards Perfect Induction as 'formally valid' induction. An induction, on the other hand, is said to be *imperfect* when the universal conclusion is reached after a survey of only some of the instances, as when we say all the students of this class have prepared their lesson from a random examination of many of them. In an imperfect induction there can never be perfect assurance owing to the more or less precarious extension to all cases of what has been observed in but a few or many. Thus, what is called imperfect induction really illustrates induction proper—the leap from the known to the unknown, the passage from 'some' to 'all'; while what is called perfect induction is scarcely an induction,—it is induction improperly so called, because it is a mere summation of known facts. Hence, Mill denies Perfect Induction the status of Induction at all.

Perfect Induction is based on an observation of all the instances coming within its range;

while *Imperfect Induction*, on an observation of some of the instances.

Imperfect induction is really induction proper.

It may be mentioned in this connection that the terms perfect and imperfect induction have

The distinction between

Perfect and Imperfect Induction is, however, not uniformly drawn by all writers.

not been used uniformly by all writers. The above distinction was drawn by the scholastic logicians. Some modern writers, however, have used 'perfect induction' even in the sense of scientific induction or induction proper, while others have used it to cover all instances where there is room for certainty as distinguished from those where there is room only for probability.

Fowler.

Fowler, for example, writes, "It sometimes happens that though we may be unable to establish a fact of causation between two particular phenomena, we may be able to show that some one phenomenon stands in a causal relation to some one or other of a definite number of other phenomena. Thus, supposing a vegetable to be transplanted to a distant part of the world, we may be able to assure ourselves, by excluding other causes of difference, that any new qualities which it may assume are due either to difference of climate or to difference of soil, or to both of these causes conjointly, though our knowledge may not enable us to assign amongst these alternatives the particular cause or combination of causes to which the effect is due. Now ought such an Inference to be classified as a perfect or an imperfect Induction? If we content ourselves with stating the alternatives, the inference should be regarded, so far as it goes, as a Perfect Induction; for within the limits stated the conclusion may be considered absolutely certain. But, if, on any grounds, we suppose one of these alternatives to be more probable than the others, and we state this as our

conclusion, the inference is, of course, only a probable one, and should rank as an Imperfect Induction. The same remarks will apply to those cases in which there is any uncertainty as to the nature of the fact of causation. If the inference be, say, that the two phenomena either are one cause and the other effect, or stand to each other in the relation of cause and effect, though we may be unable to determine which of the two is cause and which is effect, or are joint effects of the same cause (adding any other alternatives which the particular case may require), the inference is, so far as it goes, a Perfect Induction. But, if one or some only of these alternatives be selected, on any grounds short of absolute or moral certainty, to the exclusion of the others, the inference is only probable, and must be regarded as merely an Imperfect Induction." (*Inductive Logic*, pp. 222-223).

§ **5. Processes Simulating Induction.** → omitted Under this head come all those cases where we apparently generalize a law from the observation of a few instances, without really doing so. The following cases may be considered in this connection :—

(1) Perfect induction, being but a summary of the instances observed, is not truly an induction.

(1) What has been described above as perfect induction is not really an induction, in as much as there is no advance in knowledge in such a case. Suppose we argue thus : The Calcutta University, the Patna University, the Dacca University, the Bombay University, the Madras University, the Allahabad University, the Punjab University the Lucknow University, the Benares Hindu University, the Delhi University, and the Mysore University

aim at the due education of Indian students; but these Universities are all the Indian Universities: therefore, all the Indian Universities aim at the due education of Indian students. Here the conclusion merely sums up what is said in the premises, without giving us any new information. It is but a compendious form of the original statements and not properly an induction.

Brief but comprehensive

The same remark applies to some forms of mathematical reasoning.

An analogous case is found in some forms of mathematical reasoning. When, for example, observing that a straight line meets the circle, the ellipse, the parabola, and the hyperbola only in two points, we conclude that the same thing is true of all conic sections, we cannot be said to reason inductively, since the conclusion only states briefly what is said separately in the premises.

(2) The proofs of Euclid, though apparently inductive are not really so, since they are but deductions from definitions, axioms, and postulates.

(2) The proofs of Euclid are sometimes regarded as Inductive : a certain truth is proved by reference to a particular diagram ; and subsequently it is generalized and believed to be true in every similar case. The angles at the base of an isosceles triangle, for example, are proved to be equal with regard to a particular diagram drawn on paper or board ; and what is found to be true in the particular case is next generalized with regard to all similar cases, *viz.*, all isosceles triangles. Mill calls such a process of mathematical reasoning '*Induction by Parity of Reasoning*' on account of the similarity (parity) of the character of reasoning in all such arguments. He, however, puts it aside as an 'apparent induction.' The argument apparently has the semblance of Induction, in as much as we proceed from the

particular to the general. When we closely examine the character of proof in such cases, we find that it is essentially Deductive. The proof is couched in general language and is applicable to this, that, or any other diagram. The diagram is meant merely to illustrate the proof which is essentially general and deductive in character. The proof really establishes a conclusion from definitions, axioms, postulates, and the truths established in the previous propositions. In such proofs, as Mill says, "the characteristic quality of Induction is wanting, since the truth obtained, though really general, is not believed on the evidence of particular instances." Thus, we see that Euclid's proofs are not Inductive but Deductive.*

(3) The determination of the orbit of a planet from some of its positions is really a deductive inference, as it follows from a knowledge of the properties of a curve.

(3) When on the observation of certain positions of a planet its orbit is discovered, the inference is taken by some to be essentially Inductive. When, for example, noticing the successive positions of Mars, we infer that it moves in an ellipse, it is contended that we proceed from the known to the unknown, from the particular to the general; and this is an important character of Inductive Inference. But we should remember that here we seemingly proceed from the particular to the general, while really we proceed from the general to the particular. If one be not familiar with the properties of an ellipse, then he can never possibly surmise

* When, however, a geometrical truth is established by a careful observation of several particular instances, the process is to be viewed as inductive and not deductive, as when we generalize that the angles at the base of an isosceles triangle are equal after measuring and comparing such angles in many cases. It then illustrates Induction by Simple Enumeration. (*Vide* Chap. XXII, § 2.)

that the orbit of Mars is elliptic by observing some of its positions. In discovering the orbit, what we really do is to deduce the character of the orbit in the particular case from our general knowledge of the properties of the corresponding curve. The argument is thus really Deductive and not Inductive.* (*Vide* Chap. XVI, § 5, Colligation of Facts).

§ 6. **The Inductive Syllogism.** Two ineffectual attempts have been made to reduce Induction to the Syllogism. Let us consider them separately.

(1) Aristotle tries to reduce an inductive inference into a syllogism of the third figure;

(1) Aristotle tries to resolve an Induction into a Syllogism of the third figure thus :—

'Henry, Smith, Thomas, and others are mortal' ;
'Henry, Smith, Thomas, and others are all men' :
Therefore, 'All men are mortal'.

This syllogistic form is described by Aristotle as "proving the major term of the middle by means of the minor." Here the minor, middle, and major

* Dr. Venn observes on this point : "The only facts which the example supposes Kepler to have had before him were a finite number of observed positions, and these he had somehow to fill in. Now, as every mathematician knows, given any number of points we can conceive as many curves as we please each of which shall fulfil the condition of passing through all these points. The true path therefore was in no way given to observation in the sense that it only required to be recognized and named : it had on the contrary to be selected or guessed from amongst the infinite number of possible curves. If it were worth going into further detail it might easily be shown that 'induction' in both senses was involved. Not only was the constructive or originative element demanded in a high degree (thus constituting the process an induction in Whewell's sense), but there was also that of generalization (thus constituting it induction in Mill's sense). What Kepler did was, from a finite number of observed positions to frame a rule for inferring all the intermediate unobserved positions, as well as those at any past or future time." (*Empirical Logic*, p. 354.)

terms are to be understood in denotation. The predicate of the conclusion ('mortal') is of the widest extent and may thus be said to be the major term. The subject of the conclusion—'men'—is of medium extent and may thus be said to be the middle term. The term which is subject in each premise is of the least extent and may thus be said to be the minor term. Hence in the syllogism the major is proved of the middle by means of the minor.

But the attempt of Aristotle may be taken as futile. When, for example, we take 'Henry, Smith, Thomas, and others are all men,' do we examine every human being? If so, the argument is not at all Inductive, for the general conclusion is then merely an epitome of the instances observed. We have seen that the essence of Induction lies in the leap from the known to the unknown which is possible only on the discovery of a general law. If there is no such leap here, there is no Inductive Inference; and so Induction is not reduced to Deduction (Syllogism). If, again, the given minor premise involves a leap from the known to the unknown, it can be effected only by a truly Inductive method and not by means of Deduction.

but the attempt is futile.

Again the Inductive Syllogism of Aristotle may be said, strictly speaking, to violate the rules of the syllogism. If we consider the syllogism, given above, we find that the copula 'are' in the minor premise means 'make up' or 'constitute'. The syllogism, accordingly, implies that each of the persons in the major premise is 'mortal' (the

major term), while the persons in the minor premise collectively constitute the class 'men' (minor term). Thus, the middle term 'Henry, Smith, Thomas and others' is used collectively in the minor premise and distributively in the major. Therefore, the conclusion 'All men are mortal' is *deductively* invalid as the syllogism involves the fallacy technically known as *the fallacy of composition.*

(2) The attempts of Aldrich and Whately to resolve induction into a syllogism are likewise unsuccessful.

(2) Aldrich and Whately try to resolve an Induction into the Syllogism thus :—'The men whom I have observed and the men whom I have not observed are mortal.' But 'All men are the men whom I have observed and the men whom I have not observed.' Therefore, 'All men are mortal.'

It is argued here that a universal conclusion may thus be arrived at by the syllogistic process. But the question at issue is—What is the ground of the major premise? We might have observed some men to be mortal; but what right have we to assume, without the help of Induction, that the men whom we have not observed are also mortal? The unobserved instances can be included in the scope of a proposition only when we take for granted the truth of the Inductive process.

An inductive argument may, however, be reduced to the syllogistic form with the uniformity of nature as major and

An inductive argument may, however, be reduced to the syllogistic form with the Uniformity of Nature as the major premise and the facts observed as the minor. (*Vide* Chap. XVIII, § 1.) Thus, 'John is mortal, Smith is mortal, Brown is mortal, Henry is mortal, therefore all men are

mortal,' may be reduced to the syllogistic form thus :—

the facts observed as minor premise.

What is true of several members of a species under certain conditions is true of all the members under the same conditions ;

Mortality is true of the several members John, Smith, Brown, and Henry of the species man under certain conditions, *i.e.*, when human nature is present in them :

Therefore, mortality is true of all members of the species man under the same conditions ; *i.e.*, all men are mortal.

Inductive generalizations form the premises of syllogisms, which interpret them by reference to particular cases.

§ 7. **Relation of Induction to Deduction.** We have seen that, in every valid syllogism, at least one of the premises must be universal. The function of the syllogism thus appears to be to interpret a universal proposition by reference to a particular case. When a universal proposition is thus assumed as a premise, it may be regarded as the outcome of prior induction. The way, then, in which we usually argue is first to arrive at a universal proposition by Induction and then to apply such a proposition to a definite or particular case. This explanation implies that Induction usually precedes Deduction. And, no doubt, as a matter of fact, universal propositions are mostly established by the application of the Inductive Canons or Methods ; only a few fundamental principles are assumed as axiomatic in Logic. But, if the question is raised as to the relative priority of Induction or Deduction, the question is not quite

Conflicting views of the relative priority of Induction or Deduction.

easy to answer. In fact, different views have been held on this point :—

(1) Some (*e.g.*, Mill) hold that Induction precedes Deduction ;

(1) Some (*e.g.*, Mill) maintain that Induction must always precede Deduction. We first arrive at universal propositions by Induction ; and next we apply these propositions to particular cases by the Deductive method.

(2) while others (*e.g.*, Jevons) contend that Deduction underlies Induction.

(2) Others (*e.g.*, Jevons) contend that Deduction precedes Induction. The supporters of this view hold that the universal proposition is first suggested to the mind by imaginative insight as a hypothesis ; and, when this hypothesis is verified, it is regarded as an Inductive Generalization. Verification (*i.e.*, deduction of facts from a universal proposition or supposition) thus precedes Induction. To arrive at an Inductive Generalization, we must proceed, according to this view, thus :—(1) Observation, (2) Analysis, (3) Elimination, (4) Hypothesis, (5) Verification (Deduction), (6) Induction.

The latter view seems to be plausible.

If we are to choose between these two views, the latter view seems to be plausible, so that in theory Deduction may be said to precede Induction. When from the observation of a few cases we pass to a general truth we can do so only by imaginatively connecting all such cases together and bringing them under a comprehensive formula applicable to them all. We never observe the general truth floating in the air, as it were. We merely guess it at the outset ; and it is only after verification that we accept it as a law governing facts. Thus, deduction, in the form of verification

seems to precede the inductive enunciation of a law or general truth. But, we should remember that this deductive procedure is generally implicit and vague. In all difficult cases, we apparently employ first the inductive canons to arrive at correct generalizations, which subsequently become the starting points of further deductive applications and exemplifications. (*Vide* Chap. XVIII, § 1).

§ **8. Induction as an Inverse Process.** Induction is sometimes described as an inverse process of deduction, as finding out from a few concrete cases a law or general truth from which such cases may be deduced. Instead of proceeding, as in the syllogism, directly from a general truth to a case illustrating it, we have in induction first to surmise such a truth from the observation of instances and then to deduce them from it for its verification before accepting it as a reliable generalization. A syllogism, as we have seen, may be likened to a hypothetical proposition, in which we proceed from the antecedent to the consequent, and not *vice versa*. In induction, on the contrary, we first pass from the consequent to the antecedent and then back again from the antecedent to the consequent in order to be sure that the supposed antecedent is really a ground adequate to account for the consequent. To explain some facts we are led to frame some hypothesis about a cause which is believed to be adequate to account for them; and, to be sure that this hypothesis is correct, we put it to test and see whether the result agrees with the facts to be explained. If this be the case, the

Induction is viewed as an inverse process of deduction, since a correct generalization can be reached only by the deductive verification of a previously suggested hypothesis.

hypothesis is taken to be an inductive generalization. Thus, if deduction is a straight and forward movement, induction may be viewed as a circuitous movement in which we move first backwards to reach a general truth by way of conjecture or hypothesis and then move forwards to verify it and so to establish it as a well-grounded generalization. (*Vide* Chap. XIX, § 9.) There seems to be a good deal of truth in this view of induction, which is advocated by Jevons, Sigwart, Bosanquet, and others in opposition to what is taught by Bacon, Mill, and Bain. Jevons, accordingly, observes—"It is the inversion of deduction which constitutes induction". (*Principles of Science*, p. 12.)

It may, however, be mentioned here that Deduction and Induction are finally but two aspects of a fundamental inferential process which aims at systematizing or rationalizing experience. As rational creatures we can never be satisfied with detached facts ; we ever try to weave them together in a connected system by means of laws. Thus, an attempt at systematization may proceed either from the side of facts or from the side of laws : we may start with facts and try to discover the thread or law connecting them ; or we may start with laws and aim at their exemplifications. In the former case, no doubt, we have first to suppose a law and verify it by deduction of facts, before accepting it as a correct generalization. But our aim in both the cases is the same, *viz.*, to organize experience.

§ **9. Induction or Deduction as a Converse Process.** Of the two processes of in-

ference, Induction and Deduction, one is sometimes said to be the converse of the other. Such a description, however, is only partially true. In Induction we pass from the particulars to the universal, from the less general to the more general, while in Deduction we pass from the universal to the particular, from the more general to the less general. But besides this contrast, there is another important point of difference between them which renders them quite distinct. In the former we aim at material truth and so examine carefully the validity of the data by an appeal to observation and experiment, while in the latter we aim only at formal truth and so accept the data as they are supplied to us. If in the one case by careful observation, variation of circumstances and elimination of accidental features we try to determine the material factors which are causally connected justifying generalizations in conformity with facts, in the other case we take for granted the premises furnished by the different branches of knowledge on their authority and only draw a conclusion which follows necessarily from the data according to the fundamental principles of consistency. Thus, the view that the one process is merely the converse of the other is only superficially true.

To describe induction or deduction as a converse process is only partially true.

§ **10. Importance of Induction.** Induction, as we have seen, enables us to generalize a truth from the observation of some individual instances. Such a generalization is important in many ways :—

(1) We thus discover the laws of nature and bring together several phenomena which would

(1) Induction reveals the laws of

Nature, thereby unifying phenomena.

otherwise seem detached. Bacon conclusively proved the importance of Induction for scientific inquiry.

(2) It is an aid to memory.

(2) We can thus remember the several instances far more easily than otherwise we may be able to do. It is not practicable for us to remember the numerous individual instances, illustrating a law, one by one. If, however, a law is established by Induction, we can remember, as it were, in a nutshell all that is involved in the law.

(3) It is a guide to future inquiry.

(3) The law or universal proposition serves as a formula or guide for future investigations. Subsequently the law may be applied to new concrete cases yielding a conclusion not suspected before.

(4) It finally establishes the inner unity and harmony of the universe.

(4) When, by careful and continued inductive research, we trace the several laws of Nature to a few ultimate laws, closely connected with one another, we discover the inner unity of the universe and understand its true character as a harmonious whole. The law of Conservation of Energy as the highest inductive generalization reveals the fundamental unity and close correspondence of the different departments of Nature.

(5) It is practically useful as indicating future issues.

(5) Induction is of great practical value, as it enables us to interpret the unknown by reference to the known and thus to secure results which may be of great moment to us. Without general laws to guide us, we cannot employ even deduction to determine an issue in a concrete case.

§ 11. **Exercises.**

1. Indicate the relation of Induction to Deduction, determining their place in scientific investigation.

2. Distinguish between perfect and imperfect induction. Which of them illustrates the true form of inductive inquiry?

3. Explain the characteristics of Induction, illustrating your remarks by examples. 'Induction by Parity of Reasoning is improperly called Induction'. Why?

4. Distinguish Induction proper from processes simulating Induction. Are the proofs of Euclid inductive generalizations? If not, when can geometrical truths be so regarded?

5. Explain and examine the view of Aristotle that "Induction is proving the major term of the middle by means of the minor".

6. Is it possible to reduce Inductive Reasoning to the Syllogism? Explain and examine the views of Aldrich and Whately.

7. Induction is sometimes described as the inverse process of Deduction. How?

8. Point out the importance of Inductive Reasoning. Who is the founder of Inductive Logic?

9. "Induction is the process of establishing general propositions, and deduction is the interpreting of them". Explain and illustrate this. Is the theory of reasoning here implied admitted by all logicians? If not, what other theory has been held?

10. You draw an isosceles triangle on a board, and prove that its two basal angles are equal, and then draw the conclusion that all isosceles triangles have their basal angles equal: explain the logical character of this conclusion.

11. Have the inductive and deductive processes of reasoning anything in common? What is common to them? In what do they differ?

CHAPTER XVI.

THE INDUCTIVE PROCESS.

As the phenomena which we have to observe are more or less complex, inductive generalizations always involve analysis and abstraction.

§ 1. **Complexity of Phenomena.** We have seen that Logic has nothing to do with an inquiry into the real character of things (which comes within the province of Metaphysics); it is concerned only with facts or phenomena as they are presented to our mind. (*Vide* Chap. III, § 4.) Most of the facts so presented are, however, of a complex nature; so that they must be broken up or analysed into their constituent elements, before we can arrive at a universal proposition. Given any object, say, 'man' or 'metal', we can form universal propositions about it in various ways by dwelling on its different features. We may, for example, say 'men are mortal,' 'men are imperfect,' 'men are intelligent,' 'men are social beings,' 'men are created,' and so on. Similarly, we can say that 'metals are heavy,' 'metals are elements,' 'metals are extended,' 'metals are material bodies,' and so forth. All these general statements imply that we select some one feature from among many, which becomes the ground of inference in any case. Inference, as we have seen, is always based on similarity (*Vide* Chap. IX, § 1); and this is prominently illustrated in Induction. To generalize is to select a common feature, which is viewed apart from other factors going with it. And,

considering the complexity of most of the facts of experience, we may form, with regard to any object or event, numerous universal propositions by reference to its different aspects or relations.

The aim of induction is to establish useful real propositions by reference to fundamental or deep-seated connections.

Though, however, theoretically we may form innumerable universal propositions with regard to any case, yet all of them are not practically useful. That 'metals are material bodies,' that 'men are not horses,' that 'umbrellas are extended' are general propositions, no doubt; but they are of no practical utility. Thus verbal propositions, as merely unfolding the meanings of the subject, are practically useless; and so they are excluded from Induction proper. Induction, as we have seen, is a real proposition, based on observation and applicable to all facts of a kindred nature. But all real propositions are not equally important or useful always. That 'gold is malleable,' that 'it is ductile,' that 'it is found in the Transvaal' are all real propositions; but they are not equally useful in every case. That 'quinine cures ague' and that 'quinine is bitter' are both universal propositions; but they are not equally important always. Hence the business of true induction is to employ special means for discovering deep-seated relations or connections of things and phenomena for definite purposes.

Induction aims at establishing universal real propositions.

Universal propositions express any one of three relations:

§ 2. **The Inductive Problem.** We know that an Induction is a universal real proposition based on observation and in harmony with the Uniformity of Nature. In every case of induction we generalize a relation between two notions or facts. Now, the possible relationships which are

generalized are ultimately resolvable into three, namely,—(1) Co-existence, (2) Succession, and (3) Equality or Inequality. All possible generalizations must involve a reference to some one of these three forms of relation.

(1) co-existence, (2) succession, and (3) equality or inequality.

(1) With regard to the relation of Co-existence, whether in time or space, it may be mentioned that there is no ground for proceeding from the known to the unknown, which is the essence of all Induction. We may, for example, observe two things or qualities together, but if we cannot trace the relation of co-existence to causation, we can never generalize the connection from the observation of a few instances. Hence is it that Bain remarks, "There is a blankness of resources in regard to the proper laws of Co-existence; their Logic is speedily exhausted." (*Induction*, p. 10.)

(1) Bare co-existence does not justify the inductive leap from the known to the unknown.

(2) A relation of Succession may be either variable or invariable. (*a*) Variable succession can evidently never constitute the ground of Inductive generalization: what varies from instance to instance can never possibly be generalized or pronounced as uniform. (*b*) Invariable succession is especially illustrated in Causation: the cause of an event is its immediate, unconditional, and invariable antecedent. The cause invariably precedes the effect. If, in certain instances of invariable succession, we discover the causal connection, then evidently we can generalize the relation, relying on it. The Inductive Problem is thus practically restricted to the discovery of the causal connection of facts or phenomena.

(2) Succession is either (*a*) variable or (*b*) invariable. (*a*) Variable succession can never be a ground of generalization. (*b*) Invariable succession, involving causation, is the ground of true Induction.

(3) It may be mentioned here that the relation of Equality or Inequality, which constitutes the proper subject-matter of mathematics, follows in most cases from mathematical axioms. Hence such a generalization illustrates rather the Deductive Method. If, however, it be contended with the empiricists (like Mill, Bain, and others) that the mathematical axioms are themselves the outcome of generalization from experience, then such generalizations must necessarily be as much uncertain as generalizations resting on co-existence. By observing the relation of equality or inequality in some cases, we are not quite justified in generalizing such a relation, when we are not aware of a causal connection. Thus, the Inductive Problem, as mentioned above, is confined to the establishment of the causal connection alone.

(3) A relation of equality or inequality is either a deduction from axioms or is precarious like relations of co-existence.

The inductive problem is thus restricted to the discernment of the causal relation.

Though the Inductive Problem is thus limited to the establishment of causal connection among phenomena, yet we find that it may be presented in two different forms : (*a*) either a cause may be given, and we try to find out its effect ; or (*b*) the effect may be given, and we try to discover its cause.

The inductive problem wears two forms : (*a*) an inquiry from cause to effect, or (*b*) an inquiry from effect to cause.

(*a*) In the first form of inductive inquiry we may by direct observation or experiment find out the effect which follows from the given cause and thus generalize the relation between them. Of course, it should be remembered that, in dangerous cases, it would not be wise to have recourse to experiment. In such cases, we should mainly depend on observation ; and, in complex instances, we may have recourse to the Deductive Method,

(*a*) In the first form, we may often find out the effect by experiment.

by calculating the consequences which are likely to follow from the given causes.

(b) In the second form, we suppose a cause and then see by the employment of the experimental methods whether the effect follows from it.

(*b*) In the second form of inductive inquiry, it is not possible for us to go backwards and thus to find out the cause which actually gave rise to the effect. As Mr. Read puts it, "The past cause can never be recovered either by Nature or by Magic". In such cases we are driven to suppose a cause which might have produced the effect. The supposition or hypothesis, as it is called in Logic, is next tested by the Inductive Canons with a view to verification. Thus, though the Inductive Problem may be presented in either of two forms, yet its solution must always be in one direction, from cause—real or supposed—to effect.

In solving an inductive problem we thus always proceed from cause (given or supposed) to effect.

omitted →

§ 3. History of the Inductive Problem.

We have seen in the last section that Induction aims at arriving at a universal proposition which extends beyond the range of our particular experience and is more than a mere probability. It enquires how from one or more cases we come to a law which holds good in all cases of the same kind. When we look back for its history we find Aristotle describing it as a process of ascending from the particulars to the universal. He says that we establish a universal proposition by reference to all the particular instances illustrating it. According to him to show that an Induction is formally valid all the instances under it must be cited. This he considers possible with the help of a syllogism, commonly known as Inductive Syllogism, *e.g.*,

Aristotle.

The cow, the buffalo, the sheep, etc., ruminate,
The cow, the buffalo, the sheep, etc., are horned animals,
∴ All horned animals ruminate (*Vide* Chap. XV, § 6). This kind of inference, called by Aristotle Induction, is usually known as Perfect Induction and is really deductive rather than inductive.

The Scholastic and Mediæval Logicians used Induction to mean a summation of particular instances, *e.g.*, concluding a certain class of things to possess a certain quality after finding it in all the members of the class. Such an inference was called by the Scholastics Perfect Induction as distinguished from what they called Imperfect Induction where a universal conclusion is drawn on an observation of some like instances. (*Vide* Chap. XV, § 4). Thus, according to them the essence of Induction consists in the enumeration of instances. Now, the so-called Perfect Induction cannot properly be called an Inference which implies a progress from the known to the unknown as in it the conclusion is merely a re-assertion of the premises, a short-hand registration, as it were, of observed facts. The so-called Imperfect Induction also is uncertain due to its exposure to the risk of a contradictory instance. Moreover, these inductions cannot yield universal propositions which imply necessary or causal relation. Hence the opposition of Bacon and Mill to such kinds of Induction. Scholastics. Bacon.

Bacon said that Induction cannot be entirely a process of enumeration which "is a puerile thing

exposed to the danger from a contradictory instance." The object of Induction is the attainment of the knowledge of causes which is the basis of all true generalizations. The methods of such Induction are to be found in his celebrated work the *Novum Organon* where he describes the acquisition of all knowledge as the process of "interpreting nature." Thus, in order to understand Nature, we should observe her ways of action. We should observe the facts as they appear with a mind free from all "Idola" or prepossessions and collect a large number of instances from which general laws might gradually be gathered. (*Vide* foot-note to § 1, Ch. XXXI). His Method thus aims at strict fidelity to facts by allowing facts to speak for themselves. The salient features of his inductive method, sometimes called Baconian Method, are : (1) collection of a large number of instances of the phenomenon under investigation ; (2) exclusion from them by comparison of all elements that do not accompany the phenomenon ; and (3) detection, as a result of the elimination of the non-essential, of the "form" or cause of which the phenomenon is the effect. Thus we see that we owe to Francis Bacon the rudiments of the Method of Scientific Induction, subsequently elaborated by J. S. Mill, though, however, more than three centuries earlier Roger Bacon indicated the importance of experiment in scientific investigation.

We have seen that with Bacon observation of Nature (facts) was the sole ground of Induction.

But mere observation is not sufficient to give us a knowledge of causal relation for the attainment of which we should exclude the inert factors and select the potent. This led Mill to formulate the general Methods of Induction which are but methods of elimination deducible from the Law of Causation. (*Vide* Chapter XVIII, § 1.) Again, with Bacon Induction aims at rather *discovering* causal connections than proving them. But Mill defines Induction as "the process of *discovering* and *proving* general propositions." The main steps in his inductive procedure are: (1) observation of facts, (2) analysis of a complex phenomenon into antecedent and consequent, (3) exclusion of the circumstances which are not invariably present with the phenomenon under investigation with the help of the methods of elimination, (4) establishing causal connection between the antecedent and consequent uniformly related by way of sequence, and (5) verifying or proving this causal connection by a reference to actual facts.

Mill.

Before leaving this section we may have a brief reference to the contributions of W. Whewell to Inductive doctrine. According to Whewell the whole process of establishing general propositions is Induction. As against Mill's empiricism that our knowledge is entirely made up of sense-impressions he indicates the importance of thought which renders knowledge possible by unifying the discrete sense-impressions with the help of 'Ideas' or concepts. Particular facts of experience by themselves cannot give us knowledge: they are

Whewell.

to be bound together or unified by appropriate concepts or ideas. This process of unification of particulars he terms *Colligation* which according to him is the essence of Induction. Thus he describes Induction as "the *process* of a true Colligation of Facts by means of an exact and appropriate conception." And he says "The grand problem of Science is to superinduce Ideas or Conceptions upon Facts. The business of the discoverer after familiarising with facts, is to compare them with conception after conception, in the view of finding out after a longer or shorter process of trial and rejection, what conception is exactly 'appropriate' to the facts under his consideration. When the investigator has at length by a happy guess, hit upon the appropriate conception, he is said to 'Colligate' the facts, to 'bind them into a unity'." According to Whewell, therefore, Induction is a method rather of *discovery* than proof, as it aims at discovering the concepts which would bind together particular facts of experience.

§ 4. **Conditions of Induction.** The conditions of Inductive Inquiry are (I) partly subjective and (II) partly objective. We may briefly indicate the conditions thus :—

I. Subjective conditions :

(I) The **subjective conditions** include the following :—

(1) Patience and perseverance.

(1) Patience and perseverance in research. Observation or Experiment is seldom successful in only a few cases. There may be distracting circumstances or disturbing agencies which may baffle inquiry, unless it is steadily pursued in a

definite direction with care and patience. The lives of scientists often illustrate how by perseverance they finally succeeded in their investigations, though they had on many occasions been disappointed before.

(2) Absence of bias or prejudice.

(2) Inductive inquiry must always be carried on in an impartial and unprejudiced way. As Bacon has shown, the mind must be freed from the Idola before it can reflect like a mirror the correct state of things. (*Vide* Chap. XXXI, § 1, foot-note.) We often imagine that certain qualities or relations are present in a certain case because we are anxious to find them there. Pope observes—

"Trace science, then, with modesty thy guide ;
First strip off all her equipage of pride :
Deduct what is but vanity or dress,
Or learning's luxury, or idleness".

(3) Concentration, abstraction, and well-regulated imagination.

(3) Inductive inquiry requires further the power of close application and high abstraction, so essential to the generalizing process. Without reflective analysis and imaginative insight, inductive inquiry often becomes unavailing. "It is not", says Venn, "simple generalization, in the sense of mere extension, which we have to perform, but generalization through a judicious use of exclusions resting on observation and experiment". (*Empirical Logic*, p. 352.) Hence the importance of hypothesis which as a tentative conception guides the selection of instances in an inductive inquiry and determines the line of observation and experiment. (*Vide* Chap. XIX, § 9.)

(4) Belief in Causality and Uniformity.

(4) Belief in Causality and Uniformity without which there can be no extension from some cases to all. Whether actually there is a ground for such belief or not, we need not consider here; but without the belief we can never take a step from the known to the unknown.

II. Objective conditions :

(II) The **objective conditions** of Induction may be indicated thus :—

(1) Separation of elements.

(1) We have seen that the facts or phenomena observed are generally of a complex character. In order, therefore, to discover the cause or the effect of any event, we must isolate it from its natural concomitants which go to disguise it or modify its character. In determining, for example, the effect of a medicine, we should exclude all those consequences which are known to follow from climate, habit, diet, or individual constitution in any case. Similarly, in investigating the cause of a malady, physical or mental, we should separate all those circumstances which, though accompanying it, are also found without it. Thus, in inquiring into the cause of plague or of insanity, we overlook the character of weather or of diet which may vary without affecting the disease. If, as a matter of fact, we find plague to be ordinarily connected with locality, and insanity with marriage between near consanguineous relations, then we separate these circumstances as probable causes and concentrate our investigations on them. We find, accordingly, Analysis* playing a prominent part in all

* Analysis, it should be remembered, wears two distinct forms, known as (1) logical or psychological and (2) physical or chemical.

inductive inquiries. We must at the outset resolve or analyse a complex fact or phenomenon into its constituent factors before we can expect to discover its true cause or effect.

(2) Elimination.

(2) These remarks also make it clear that Elimination, or exclusion of the non-essential factors, is also an important step in the inductive inquiry. If, in the above illustration, we fail to leave out the effects of climate, habit, diet, personal constitution, or weather, then we cannot satisfactorily determine the true effect of medicine in any case or the proper cause of plague or insanity. "We know", says Bain, "from the law of Causation that in the changes going on in the world, the present situation is the result of the previous situation; and if that previous situation were reproduced so would

(1) *Logical analysis* is the ideal separation of elements which cannot actually be isolated, as when we analyse a ball into its constituent qualities (roundness, hardness, weight, *etc.*). This form of analysis is effected by the successive direction of attention to one feature at a time and the withdrawal of attention from the other features for the time being, thus exhaustively considering all the constituent elements or features. If, instead of considering all these elements or features, we only dwell on some, overlooking the rest, then what we call 'abstraction' is illustrated. Thus, when, in the case of a ball, we take into account simply the weight or form, ignoring the other qualities, we may be said to exercise abstraction. Abstraction may therefore, be called incomplete or imperfect analysis, and analysis, thorough-going abstraction. Logical analysis should be distinguished from physical. (2) *Physical analysis* implies the actual separation of elements or parts, as in what we call chemical analysis. It is illustrated in inductive inquiry when we really separate a supposed cause or effect from its attendant circumstances in order to determine its proper logical value. If, for example, we separate an article of diet from the other articles of food, with which it is usually taken, in order to estimate its effect on health, we have recourse to this form of analysis. Both these forms of analysis are illustrated in inductive investigations: the logical or psychological form, usually known as *resolution*, is included in the subjective conditions, while the physical or chemical, underlying *elimination* in the objective. Synthesis in every case tests the correctness and adequacy of prior analysis. (*Vide* Chap. XXVIII, § 2.)

the present. But this is not all; for we may be able to show that if a *certain part* of the previous situation were reproduced, the present would follow; we can put aside all otiose or inert accompaniments and reduce the antecedent circumstances to those really operative. This is the process of Inductive Elimination, required alike in special and in general inquiries as to cause and effect." (*Logic*, II, pp. 41-42.) We should remember that Elimination always presupposes analysis: without analysis, as shown in the preceding paragraph, we can never eliminate and so we cannot correctly generalize. All the inductive methods, underlying the generalizing process, involve, therefore, analysis and elimination in a more or less prominent form.

The Inductive Methods always involve analysis and elimination,

which require variation of circumstances.

We must remember in this connection that, in order successfully to employ analysis and elimination for inductive generalizations, we must *vary the circumstances*, so that the phenomena connected by way of causation may be singled out and the accidental accompaniments, rejected. As in nature several factors usually go together, which it is impracticable to study apart by complete isolation, it is often difficult to distinguish the true cause or effect from its accidental accompaniments. We are, however, enabled to distinguish them by **the method of varying the circumstances** which brings successively before our view different combinations, the varying factors of which we reject as accidental and the constant factor or factors we select as essential—thus taking a step towards generalization. If, for example, we never

find the antecedent *A* alone nor the consequent *a* alone, but we always find them mixed up with other antecedents and consequents, then we may succeed in discovering a causal connection between *A* and *a* when they remain constant in the midst of variations of the other antecedents and consequents, *e.g.*, when *A*BC are followed by *a*df, *A*DE by *a*bc, and *A*FG by *a*de. (*Vide* Chap. XVIII, § 2 and § 4.) Hence we see the importance of observation and experiment as the means of supplying a varied combination of antecedents and consequents, so that the inert or accidental factors may easily be excluded and the potent or material factors, found out. This is what is described by Bacon as 'varying the circumstances.' The Principle of Elimination can, therefore, successfully work when it is aided by adequate observation and experiment. This is well illustrated in the following account of an inquiry into the cause of Endemic Goitre given by Minto in his *Logic* :—

Observation and experiment are aids to elimination.

"Instances of the disease have been collected from the medical observations of all countries over many years. Why is it endemic in some localities and not in others? We proceed on the assumption that the cause, whatever it is, must be some circumstance common to all localities where it is endemic. If any such circumstance is obvious at once, we may conclude on the mere principle of repeated coincidence that there is causal connection between it and the disease, and continue our inquiry into the nature of the connexion. But if no such circumstance is obvious, then in the

An illustration.

course of our search for it we eliminate, as fortuitous, conditions that are present in some cases but absent in others. One of the earliest theories was that endemic goitre was connected with the altitude and configuration of the ground, some notorious centres of it being deeply cleft mountain valleys, with little air and wind and damp marshy soil. But wider observation found it in many valleys neither narrower nor deeper than others that were exempt, and also in wide exposed valleys such as the Aar. Was it due to the geological formation? This also had to be abandoned, for the disease is often incident within very narrow limits, occurring in some villages and sparing others though the geological formation is absolutely the same. Was it due to the character of the drinking-water? Especially to the presence of lime or magnesia? This theory was held strongly, and certain springs characterised as goitre-springs. But the springs in some goitre centres show not a trace of magnesia. The comparative immunity of coast regions suggested that it might be owing to a deficiency of iodine in the drinking-water and the air, and many instances were adduced in favour of this. But further inquiries made out the presence of iodine in considerable quantities, in the air, the water, and the vegetation of districts where goitre was widely prevalent; while in Cuba it is said that not a trace of iodine is discoverable either in the air or the water, and yet it is quite free from goitre. After a huge multiplication of instances, resulting in the

elimination of every local condition that had been suggested as a possible cause, Hirsch came to the conclusion that the true cause must be a morbid poison, and that endemic goitre has to be reckoned among the infectious diseases." (Pp. 319—320.)

(3) Verification, or confirmation of the surmise (previously formed by analysis and abstraction) by an appeal to facts, is also an essential condition of every inductive generalization. "Were all our processes absolutely trustworthy such a stage as this would not be required; but being what they are it would be rash to omit this safeguard." (Venn, *Empirical Logic*, p. 352.) If a hypothesis is thus proved to be false, then we are driven to frame a new one by fresh analysis and abstraction, until facts bear out our supposition. When a hypothesis is verified, or established beyond doubt as consistent with facts, it is regarded as a law.

(3) Verification.

§ 5. **The Inductive Procedure.** The preceding remarks must have made it clear that in order to generalize a proposition we must have certain facts. Facts, as we have seen, supply materials for generalization.

The inductive procedure involves—

(1) The first step, therefore, in the Inductive process is a patient, careful, and thorough examination of facts presented to the mind. Such observation, moreover, should not be capricious, but must be well-regulated; we are to observe the relevant facts, withdrawing our attention from those that are irrelevant.

(1) a careful study of relevant facts

(2) The second step in the Inductive process consists in the definition or clear and accurate

(2) a clear definition them,

statement of the features found in the facts observed. This process of definition excludes the possibility of confusion and misdirected inquiry.

(3) analysis,

(3) We must carefully analyse the facts observed. Most of the facts observed are complex in character; some features found in such complex facts are inert or accidental, while others are efficient or essential. To distinguish the inert or accidental from the potent or essential factors we must have recourse to analysis. Such analysis is usually in the first instance mental, suggesting lines of inquiry, and in the next physical, leading to the elimination or exclusion of accidental factors and the selection of essential ones furnishing grounds for generalization. Mental analysis enables us to form conjectures or hypotheses as to the true cause, while physical analysis enables us to test a hypothesis by actual separation, exclusion and selection, *i.e.*, by systematic elimination as involved in the employment of the Inductive Methods. (*Vide* Chap. XVIII, § 3.)

(4) framing of hypothesis,

(4) As mentioned just now, having withdrawn our attention from the inert or accidental factors, we next frame a hypothesis by reference to what appears to be potent or essential. This hypothesis is intended to explain the facts in question. It is to guide our inquiry by the selection of instances which are likely to test its validity. A hypothesis thus limits the range of observation, which is henceforth rendered still more definite and systematic, being controlled by the hypothesis.

(5) employ-

(5) When instances are thus brought before

the mind as suggested by a hypothesis, they are tested by the Inductive Canons or Rules with a view to determine its validity. These Inductive Canons are but systematic ways or forms of elimination with a view to the dicovery of the true cause or effect in any case. If observation brings before our mind instances of sequence or co-existence, elimination enables us to determine what are the essential factors in this sequence or co-existence by the exclusion of the accidental adjuncts or accompaniments. The unconditionality of causal connection is thus brought out by elimination, though mere invariability may be indicated by observation. (*Vide* Chap. XVIII, § 3.) The Inductive Canons, helping the selection of appropriate factors by means of due elimination, are, as we shall read in Chapter XVIII, five in number. They enable us to establish a causal connection when it is previously suggested by a hypothesis. As they directly follow from the Law of Causation, they have fitly been described by Bain as rather 'Deductive Methods, called by courtesy Inductive.' (*Vide* Chap. XVIII, § 1.)

ment of the Inductive Methods or the use of systematic elimination.

(6) When a hypothesis is thus found to agree with facts, it is accepted as a law. Verification finally sets at rest our inductive groping in the dark and satisfies us that the end of our inquiry is attained.

(6) verification.

§ 6. **Colligation of Facts**. Dr. Whewell regards Induction as 'Colligation of Facts.' By 'Colligation of Facts' he understands the subjective

omitted

Dr. Whewell identifies Induction with Colliga-

tion of Facts. Colligation is a subjective synthesis of facts by a suitable notion or hypothesis.

synthesis or mental union of facts or materials supplied to the mind, by an exact and appropriate conception. Thus, he uses it to mean "the act of bringing a number of facts actually observed under a general description," as, for example, by gradually going round a new country and noting its surroundings we call its shape to be round or circular. According to Dr. Whewell, whenever we arrive at a general proposition on the observation of several individual instances, we really bring together all instances of a particular class under a suitable conception or hypothesis. It is the hypothesis or notion which enables us to 'colligate' or 'bind together' (Latin *colligare,—con*, together, and *ligare*, to bind), as it were, all the facts to which the hypothesis or notion is applicable. The notion or hypothesis is supplied by the mind; and it is, so to speak, the tie or bond uniting the materials constituting the subject-matter of the generalization. A notion may thus be viewed as a string which fastens together facts in a coherent whole of experience governed by law.

Colligation is not necessarily an Induction.

Colligation of facts, however, is not the same as Induction. As Mill observes, "Induction is colligation," but "colligation is not necessarily Induction." In every Inductive generalization we, no doubt, 'colligate' or bring together facts of a certain kind; but it can never be said that whenever we bring together facts by an appropriate general conception, we also form an induction. In description, definition, or classification, for example, we 'colligate' facts by a general notion; but no

connection is proved, as is required in Induction. We have already read the difference between a notion and a general proposition. (*Vide* Chap. XV, § 2.) Even when a general notion is a complex idea, involving the co-presence of several elements, it is distinct from Induction. For, in Induction the co-presence or connection is to be *proved* by laborious research (namely, by the application of the Inductive Canons), while in a notion the co-presence or connection is *assumed*. As Mill points out, Whewell confounds Induction with mere Description. A description, by simply delineating the features of a class, brings together its members and thus 'colligates' them; but a description never proves a connection nor explains it. The essence of Induction lies in this proof or explanation. Similarly, when we connect together the places of a planet by reference to its orbit, we colligate them; but, as shown above (*Vide* Chap. XV, § 5), there is scarcely any Induction in such a process.

In induction a connection is proved, which is assumed in a notion, colligating facts.

Dr. Whewell confounds Induction with Description.

§ 7. **Scientific and Unscientific Induction.** Induction, as we have seen, is the inference of a universal real proposition from the observation of some individual instances, in accordance with the Uniformity of Nature. But such generalizations may be reached either (1) by simple observation or (2) by laborious research. (1) The one form is illustrated in all popular generalizations which merely rest on uniform or uncontradicted experience, without an attempt to verify whether there is any real ground for such inference or not. Thus,

(1) Unscientific Induction is illustrated in popular generalizations resting on mere un-

contradicted experience.

a child that has seen only black dogs may take all dogs to be black ; a foreigner, coming across only dishonest natives of a particular land, may take all the natives to be so ; the rustics of Northern Europe, having no information of the rest of the world, may think that all men are white, as some negroes are led to believe that all men are black. Men, similarly, believe that all crows are black, though albinos of the species are not altogether uncommon ; and swans are ordinarily taken to be white, though black swans are found in Australia. Our common belief in some laws of Nature (*e.g.*, that men are mortal) rests on this kind of induction. It is known as *Induction by Simple Enumeration**. As it is based merely on our faith in the Uniformity of Nature without the discovery of any causal connection, it is usually probable or uncertain. And so Bacon condemns it as "a childish thing, precarious in its conclusions and exposed to risk from a contradictory instance". The value of this form of induction we shall examine afterwards. (*Vide* Chap. XVIII, § 2 and Chap. XXII, § 1.)

It is known as Induction by Simple Enumeration

which, as Bacon points out, is more or less precarious.

(1) Scientific Induction establishes a generalization by proving causal

(2) The other form of Induction is employed in all scientific investigations ; and so it is called Scientific Induction. In it we try to place a general truth on a secure foundation by discovering some

* Induction by *Simple* Enumeration should be distinguished from Induction by *Complete* Enumeration (or *Perfect* Induction) in which all individual instances coming within a general proposition have been observed ; but the latter, as we have seen, is not properly an induction at all. (*Vide* Chap. XV, § 3.) Thus, 'All the kings who ruled in France in the eighteenth century were named Louis' may be taken as an example of Induction by Complete Enumeration.

ground uniting the facts or phenomena which constitute the subject-matter of the generalization. We shall see in the next chapter that the only secure ground of inductive generalization is found in the causal connection between phenomena, the absence of which can at most create a presumption in favour of a conjunction. Hence all the Inductive Methods, which are used for scientific generalizations, are given to deciphering such a connection. (*Vide* Chap. XVIII.) The guesses and crude beliefs of Simple Enumeration, when tested and verified by the Inductive Methods, become well-established inductions. "Such a surmise," says Fowler, "may afterwards be proved by the aid of one or other of five methods to be correct, and, in that case, it is taken out of the category of inductions *per simplicem enumerationem*, and becomes an instance of a scientific induction." (*Inductive Logic*, p. 215.)

connection by means of the experimental methods.

Induction by Simple Enumeration, when verified by the Inductive Methods, becomes Scientific Induction.

§ 8. **Complete and Incomplete Induction.** An induction may be said to be complete when we arrive at a general proposition from the observation of particular instances. It implies, as we have seen, a leap from the known to the unknown and involves a reference to causal connection. It is a universal statement which is not merely a summation of particulars. "A complete induction", says Bain, "is a generalization that shall express what is conjoined everywhere, and at all times, superseding for ever the labour of fresh observation". (*Induction*, p. 2.) This complete induction, however, may be either methodical and

Complete Induction is Induction proper in which we pass from some to all.

systematic, or immethodical and unsystematic. The former illustrates scientific induction, while the latter only induction by simple enumeration.

Incomplete Induction is used ambiguously.
(*a*) It sometimes means induction in which the causal connection is not definitely established owing to the imperfect fulfilment of the experimental methods.
(*b*) It means also induction which operates subconsciously in determining inferences in particular cases, without developing a universal proposition.

The term 'Incomplete Induction' has been used in at least two distinct senses. (*a*) Some understand by it that form of induction in which we are dimly aware of a causal connection between the phenomena under investigation, but we cannot definitely establish it owing to the imperfect application of the Inductive Methods. Thus, incomplete inductions, according to Fowler, are "the results of an imperfect fulfilment of one or other of the Inductive methods". [*Inductive Logic*, p. 234.] (*b*) Others hold that 'incomplete induction' implies that the universal proposition is not developed out of particular instances, but it operates subconsciously in the mind in determining inferences in other particular cases. When, for example, we reason from particulars to particular, we find this form of induction involved. If, after finding, say, ten mangoes of a basket to be sweet, I expect that the eleventh and twelfth mangoes are also sweet, then, according to this view, 'incomplete induction' is illustrated. My inference with regard to the eleventh or twelfth mango cannot be true unless the universal proposition 'all mangoes of the basket are sweet' be true. But this universal proposition is not explicitly formulated ; it operates only implicitly in the mind to justify a conclusion in a particular case. We shall see in Chapter XXII that analogical inference is of this character. When we draw such an inference, we are generally not

aware of any causal link; for to be conscious of such a link is to take a step towards generalization or complete induction. If we know, for example, that A is the cause of B, then surely we are led to generalize that wherever A is, there B also is present.

§ 9. **Exercises.**

1. Indicate the character of Inductive Inquiry and the different forms it may assume.

2. Determine the conditions of inductive research. What do you understand by Elimination and Varying the Circumstances? Is Elimination the essence of Induction?

3. Analyse the Inductive Process, illustrating the steps by examples.

4. What do you understand by Colligation of Facts? How does it differ from Induction?

5. Distinguish between (1) Scientific and Unscientific Induction, (2) Induction by Simple Enumeration and Induction by Complete Enumeration.

6. What do you understand by Analysis and Abstraction? Are they connected in any way with Inductive Investigation?

7. Distinguish between (1) Elimination and Resolution and (2) Complete and Incomplete Induction.

8. 'The method of Induction consists throughout in the framing of hypotheses to explain the phenomena given in experience, and the verification of those hypotheses by constant appeal to facts.'—Explain the above. How far does it agree with Mill's theory of the nature of Induction?

9. Consider whether there is any theoretical ground for the distinction, in respect to proof, between propositions of sequence and propositions of co-existence, and whether there is any practical utility in the distinction.

10. Clearly explain what is meant by the Method of Varying the circumstances and show how it helps inductive investigation. Is the Method connected in any way with Elimination?

CHAPTER XVII.

THE GROUNDS OF INDUCTION.

Inference always involves both matter and thought.

§ 1. Formal and Material Grounds. We have seen that every form of inference—nay, every product of thought—involves two elements, *viz.*, form and matter. (*Vide* Chap. I, § 8, foot-note, and Chap. II, § 1.) There are principles or ways according to which we reason and also materials about which we reason. We have further read that in the case of deductive inference, which is concerned only with formal truth, the materials or data are accepted in logic without independent investigation. In syllogism, for example, we have only to see whether any conclusion necessarily follows or not from the premises according to the fundamental laws of thought, and we have nothing to do with the material truth or falsity of the premises. In inductive reasoning, however, it is otherwise. We know that induction is concerned with material, and not merely with formal, truth. Hence, in all inductive inquiries, we must satisfy ourselves as to the actual truth of the data or premises. We also know that in induction we always proceed from particular instances to a general truth or universal proposition. The grounds of Induction, accordingly, are (I) partly formal and (II) partly material.

The matter is assumed in Deduction, while it is examined in Induction.

I. *The Formal Ground of Induction*

(I) The Formal Ground of Induction includes (1) the Law of Uniformity of Nature and (2) the Law

of Causation. (*Vide* Chap. II, § 7 and § 8.) We have seen that the invariable tendency of the mind is to generalize, to unify—even when there is imperfect similarity. (*Vide* Chap. III, § 7 and *The Elements of Morals*, Chap. XX, § 5.) Our hasty and incorrect generalizations prove but the irresistible tendency of the mind to think in this way. In order, however, that there may be a secure ground for a generalization, we must discover a causal link justifying the extension to other like cases of what is observed only in a few. The assumptions underlying all inductive generalizations are (*a*) that all the individuals of a class are characterized by the same nature or essence, so that what is fundamentally true of some can naturally be expected of all, and (*b*) that the manifestation or behaviour of a thing is always causally connected with its essential nature. Hence the Principle of Similarity and that of Ground and Consequent are regarded by realistic writers as constituting the very basis of every inductive inquiry. (*Vide* Chap. I, § 6 and Chap. II, § 4 and § 9.) The Principle of Similarity implies, as we have seen, Identity of Essence, and the Principle of Ground and Consequent implies a Causal Connection; and these are the essential formal conditions of all inductive research. They are the fundamental principles on which every form of inductive reasoning ultimately rests.

includes (1) the Law of Uniformity of Nature and (2) the Law of Causation,

which are interpreted by realistic writers as (*a*) the Principle of Similarity and (*b*) the Principle of Ground and Consequent.

(II) The Material Ground of Induction includes Observation and Experiment, which supply materials for generalization. Whenever we generalize, there must be some subject-matter; and this evi-

II. *The Material Ground* includes Observation and Experiment.

dently is furnished by Observation and Experiment. They are the sources whence the materials of all inductive reasoning are derived. Let us consider these Grounds of Induction separately.

The Law of Uniformity explains the invariable tendency of the mind to generalize,

§ 2. **Law of Uniformity of Nature.** We have already explained this law in chapter II, § 7. We need only mention here that mere uniformity without causation does not afford a justification for extending to unknown cases what has been observed in some, however much we may naturally be disposed to do so. Such a justification is found in the causal link between the phenomena under investigation. We might have noticed uniformity of certain relations in the past; but we can never with confidence extend it to the future if no causal connection is known to subsist between the related objects or features. In fact, uniformity by itself scarcely means anything: uniformity is uniformity in behaviour, nature, or action, which involves an appeal to the way in which a cause operates. Thus, the Law of Uniformity alone can never be regarded as the ultimate ground of induction; it must be taken in connection with Causation that we may be allowed to infer a universal real proposition from the observation of a few instances.

which is found to be legitimate when supported by Causation.

Causation implies that an effect is produced by a cause.

§ 3. **Law of Causation.** The law of causation strictly speaking implies that an event is produced by a cause. As Kanada says, कारणभावात् कार्य्यभावः, *i.e.*, 'from existence of the cause is existence of the effect.' (*Vaisheshika Aphorisms*, IV, i, 3. Gough's Edition, p. 133.) That the same cause always

gives rise to the same effect is not implied in causation, though it is justified by experience and supported by the Uniformity of Nature. Uniformity is thus no essential part of causality, which may operate though there may be continual surprises owing to its capricious operation. But, without uniformity, there would be no coherent experience and no room for expectation. (*Vide The Elements of Morals*, Chap. XX, § 5.) We find, accordingly, that as a matter of fact causality operates uniformly in nature, affording room for expectation and prediction. Thus, *Causation, as modified by Uniformity, constitutes the true formal ground of Induction.* And Causation, when so modified, includes, as we have seen, the Principle of Similarity and the Principle of Ground and Consequent. (*Vide* Chap. II, § 4, § 9 and § 11.)

Causation as modified by Uniformity constitutes the true formal ground of Induction.

We should remember here that the causal law assumes a mechanical or external relation between the different parts of the universe. As we are but spectators of the world, we witness various phenomena in conjunction with others; and our rational constitution leads us to construe them as a connected whole by discovering links which bind them together. Where we fail to detect a connection, we treat the relation as accidental, though really nothing is accidental in this well-ordered universe. (*Vide* Chap. XXI, § 2). Only an indwelling Spirit that pervades it through and through can read aright the connections of its diverse phenomena. As we have to survey it from outside we are constrained to infer connections by signs fur-

The law of causation as understood in science assumes a mechanical relation between the different parts of the world.

nished by experience. Thus, though as a matter of fact the effect is but the cause transformed, yet the limitations of our intelligence and the consequent imperfection of our knowledge lead us to view them as separate and then to construe them as antecedent and consequent bound together by inviolable necessity. (*Vide* § 6.)

In Logic, we are concerned, not with the question of the ultimate character of causation, nor with the way in which we come to know it, but with its marks or characteristics by which it can be recognised.

In Logic, we inquire into the character of the causal relation with a view to determine its characteristics or fundamental features. The logical aspect of this inquiry should not be confounded with the metaphysical or psychological. The metaphysical aspect of the problem implies an inquiry into the real character of the causal relation: Whether, for example, by cause we are to understand a thing, a force, or a phenomenon; and whether the causal connection is real or fictitious. Such a question comes within the province of Metaphysics and falls outside the scope of Logic proper. To determine, again, the way in which we arrive at a knowledge of the causal relation is a psychological question, which should not be confounded with the logical inquiry. In Logic, without raising a question as to the real character of cause or the source of our knowledge of it, we merely try to ascertain the characteristics or marks by which the causal relation can be distinguished from other relations. The question here is what are the tests or features by which we can distinguish a cause from an accidental circumstance. Let us inquire into this logical aspect of the question a little carefully.

§ 4. Scientific View of Causation. For the purpose of logical or scientific inquiry we consider the causal connection as existing between two facts, one of which is the *consequent* and the other is the *antecedent.*

Scientific view of Causation.

(A) When we analyse the *qualitative aspect* of the causal relation we find the following features :—

(A) *Qualitative aspect* : Causation implies

(1) The causal connection, as just now remarked, always involves a relation between two factors, one of which is regarded as the consequent (effect) and the other as the antecedent (cause).

(1) a relation of succession between two factors, of which

(2) The consequent is always regarded as an event in time, that is, a change or phenomenon to be accounted for.

(2) the consequent is regarded as the effect and

(3) The cause, which explains the phenomenon or effect, precedes it. Thus, the causal relation is a case of succession, the antecedent being the cause and the consequent, the effect.*

(3) the antecedent as the cause.

(4) But mere succession does not constitute the causal relation. Succession may be either variable or invariable. Variable succession never inspires in us a belief in causal connection. Clouds, for example, may precede or succeed sunrise ; and so we never regard the one as the cause or effect of the other. It is the invariable succession which is taken to be the mark of causation, so that we regard the cause as the invariable antecedent and the effect as the invariable consequent. Thus, the

(4) Causation is invariable succession.

The cause is thus the invariable,

* It is immaterial in Logic to discuss the question whether the cause and the effect may be simultaneous. Even if they be so, we do not as a rule apprehend simultaneously what may simultaneously happen. Thus, even simultaneous events may *seem to us* as successive.

stroke of a sword is taken to be the cause of the flow of blood; and the administration of poison, the cause of death.

(5) unconditional,

(5) But this invariable relation alone does not constitute causality. Had it been so, then night might be regarded as the cause of day, or day of night, since the one invariably precedes the other. Thus, the cause is not merely the invariable antecedent, but also the unconditional antecedent. By 'unconditional' is meant that, without any other condition, the antecedent, which is regarded as the cause, is able to lead or give rise to the effect. Thus, the sun is taken to be the cause of light, as the presence of the one is invariably and unconditionally followed by the other.

and (6) immediate antecedent.

(6) Cause, moreover, is not a remote antecedent, but the proximate or immediate antecedent which, without any other condition, brings about the effect. The cause, for example, in the above illustrations, is the stroke of a sword, the administration of poison, or the presence of the sun, which immediately precedes the effect.

If we take the above marks or features into our consideration, then we may define a cause as the immediate, unconditional, and invariable antecedent which produces the effect.

(B) *Quantitative aspect*:

The effect is but the cause transformed, and hence they are

(B) The *quantitative aspect* of the causal relation is illustrated in the fact that there is perfect equivalence between the cause and the effect in every case. Every effect is due to the expenditure of some energy. What we regard as the effect is the energy transformed; and what we regard as the

cause is the energy prior to such transformation. When, for example, an individual takes food and feels refreshed, the feeling of refreshment is regarded as the effect, which is due to the nourishment supplied by the food. The quantitative equivalence involved in the causal relation is due to conservation of energy. The **doctrine of conservation of energy** implies that—

equivalent in energy.

The doctrine of conservation of energy

(1) The total amount of energy in the universe remains constant, and no part of it can be destroyed ;

(2) One form of energy may be transformed into another (for example, the molar into the molecular, the physical into the chemical, the chemical into the vital) ;

(3) In the process of change, work is done and an effect produced ; but no energy is lost.*

The doctrine of conservation of energy has been established conclusively by the modern sciences ; and it may be proved in detail with regard to any and every subject-matter. We may illustrate the law here by an example or two. When, for example, a cannon ball strikes a wall, which is not visibly affected in any way by such impact, we may be led to think that the energy of the moving ball is altogether lost ; and the law of conservation of energy seems to be disproved by such an instance. But, if we examine the case a little carefully, we find that an effect is really produced in the form of

is supported by the sciences.

Illustrations.

*Kanada says, न द्रव्यं कार्य्यं कारणञ्च वधति, *i.e.*, 'a substance is not destroyed either by its effect or by its cause.' (*Vaisheshika Aphorisms* 1, i, 12. Gough's Edition, p. 13.)

heat. That part of the wall against which the ball strikes becomes very hot ; and the degree of heat is proportioned to the violence of impact. We know, however, that heat is but a form of motion : it involves movement of molecules, though not movement of a mass. Thus, the case in question illustrates but the transformation of molar movement into molecular movement ; and, as we have said, there is perfect equivalence in respect of quantity between the energy of the cause and the intensity of the effect. Another illustration will make the position still more clear. When, for example, we throw a ball upwards and it rests on the roof of a building, apparently the moving energy of the ball is lost. But we really find that here kinetic energy is transformed into the potential form. Matter may be found either in motion or in position. Matter in position, though seemingly quiet, is really a store-house of energy, which may be perceived on the fulfilment of certain conditions. If, for example, the roof of the building be removed, the ball will again fall to the ground with the same energy with which it moved upwards (the resistance of the air being excluded). As Bain observes, a mountain tarn, though visibly quiet, is really a source of vast energy, which may be revealed if one of its banks give way. Thus, the cause and the effect are but different aspects of a definite amount of the same energy : what we call the 'cause' is but the effect concealed ; and the 'effect' likewise is but the cause revealed. It may be mentioned here that the doctrine of Conserva-

tion of Energy lends support to the view that cause must be the unconditional antecedent of a given phenomenon, called its effect, as the theory implies that the effect is but the cause transformed, in which case there can be nothing intermediate between the two.

ad. of motion

Distinction between Kinetic and Potential Energy, between Agent and Patient, is entirely relative.

The difference between Kinetic and Potential Energy, or between matter in motion and matter in position, brings out the difference between what are called 'Agent' and 'Patient'. The terms 'Agent' and 'Patient' are entirely relative. What seems to be purely passive is really the seat of energy which may not be manifested at the time. As Mill observes, when the administration of poison proves injurious to life, the poison is ordinarily taken to be the 'Agent' and the human system, the 'Patient.' But this human system must be susceptible to the influence of the poison; otherwise it can never exercise an injurious influence on the human organism. Thus, what seems to be a passive collocation or arrangement of materials or circumstances is not really devoid of energy altogether; it illustrates but potential energy under definite conditions.

The cause or the effect may be either a moving energy, or a collocation, or a combina tion of them.

Often, for the production of an effect, a combination of circumstances or conditions is essential, some of which may be in the form of moving energy and some in the form of collocation. The lunar eclipse, for example, is produced by a collocation of the sun, the earth, and the moon, causing an interception of the solar rays. The movement of a boat down a stream is caused both by energy

Illustrations.

and collocation. Sometimes, by ellipsis, a collocation may simply be referred to as a cause. When, for example, a hydraulic press is employed in compressing bales of jute, the press is regarded as the cause, though the real moving power is found in the moving or running stream. Similarly, an effect may sometimes be represented as a collocation. When, for example, masons construct a building, the effect is illustrated, not in the form of moving energy, but in the form of a collocation, namely, a definite arrangement of materials known as a building. These illustrations bring out the importance of collocation, whether found in the cause or in the effect.

§ 5. **Causes and Conditions.** From the preceding remarks it is apparent that a cause or an effect is often a complex fact, involving several elements.

A cause is an assemblage of 'conditions'.

A cause, like an effect, may be an assemblage of several factors, each of which is called 'a condition'. As Kanada says, 'Causality results also from conjunction': 'संयोगाद्वा' (*Vaisheshika Aphorisms*, x, ii, 2. Gough's Edition, p. 305.)

Meaning of a 'condition.'

A 'condition' may briefly be defined as that which exercises some influence on the effect: the influence may be in the form of either production, prevention, or modification. Anything which helps, destroys, or retards an effect may be viewed as a *condition*. And all the conditions taken together constitute a cause. Mill thus explains the meaning of 'condition':—When, for example, an individual

An illustration.

takes mercury and goes out in cold weather and catches cold and gets fever, the cause of the

cold and fever is to be found, not merely in taking mercury, nor merely in exposure, but in both the circumstances taken together, namely, in exposure while under the influence of mercury. Each of these two circumstances is called a 'condition', and both the conditions taken together constitute the cause. It may be mentioned in this connection that conditions may be either positive or negative. A *positive* condition is that which helps or promotes the effect and so it cannot be left out without baffling the effect; while a *negative* condition is that which tends to thwart or frustrate the effect and so it cannot be introduced without defeating the effect. When, for example, a man falls to the ground through a slip of his feet, the positive condition is illustrated in the slipping of the feet, without which the effect would not have been produced. But a negative condition is illustrated in the absence of a support which might have prevented him from falling to the ground. The positive and negative conditions taken together constitute the real cause. When a condition is taken by itself, it is regarded as but a tendency. A *tendency* may thus be defined as the ability of a condition, when taken by itself, to produce an effect. And the mutual relation of the several tendencies is said to be one of *reciprocity*.

Positive and negative conditions.

Meanings of 'tendency' and 'reciprocity.'

The relation of cause and effect in complex cases is illustrated, as Mill points out, in two prominent forms :—

Causation in complex cases assumes two forms :

(1) One form is described by Mill as the Homogeneous Intermixture of Effects. In this

(1) Homogeneous intermixture

of effects, and

case, the several conditions as well as the effect are homogeneous. This is illustrated in mechanics. When, for example, two forces act on a body, the line of motion is indicated by the direction of the resultant. Here all the factors are commensurable.

(2) hetero-pathic intermixture of effects.

(2) The other form is illustrated in the Heteropathic Intermixture of Effects. Here the cause and the effect are not homogeneous, as is illustrated in chemical combination. The chemical compound in any case does not resemble the elements which give rise to it.

Whatever may be the form of causal relation, whether Homogeneous or Heteropathic, the causal inquiry must involve an examination of the antecedent circumstances prior to the effect. And though, adequately to explain a fact, it is necessary that all the conditions essential to its production must be mentioned, yet it is not always convenient to enumerate them all. Hence we find the causal conception generally interpreted from three stand-points, namely :— 1) The ordinary stand-point, (2) the scientific stand-point, and (3) the stand-point of conservation of energy. Let us consider them one by one.

Causation interpreted from three stand-points :

(1) The popular or practical stand-point.

(1) Though a cause is really a combination of several conditions, positive and negative, yet in ordinary life it is not convenient nor necessary to enumerate them all. Some of the conditions may be too well-known to require an explicit statement. When, for example, an individual falls to the ground, if we mention as its cause the force of

gravity as well as the slipping of his feet, the explanation would seem at once to be superfluous and pedantic. The force of gravity is a familiar fact known to all ; and hence only reference is made to that circumstance or condition which may not be known at the time, *viz.*, the slipping of the feet. Thus, from the popular or practical stand-point, we often mention only one or two prominent conditions, instead of all, to account for the effect. Hence, we are often led to wonder at the disproportion between some conditions viewed as cause and the greatness of an effect.

"What great events from trivial causes spring!"

Distinction between Proximate and Remote Causes.

We may notice in this connection the distinction often drawn, in the ordinary affairs of life, between what are known as Proximate and Remote Causes. By **Proximate, Immediate, or Direct Cause** is understood in common discourse the 'condition' which immediately precedes an effect, as answering questions well with regard to success at an examination, the contact of a spark with gunpowder in the case of explosion, or infection inducing illness in a certain case. It is evident, however, that none of these without other 'conditions'——such as good preparation, inflammable and expansive quality, or predisposition to a disease——can produce the effect in question. The 'conditions,' therefore, which precede the so-called proximate causes and which indirectly determine the effect are called **Remote, Mediate, or Predisposing Causes.** Sometimes the remote cause is popularly called simply 'the conditions,' while the proximate

occurance

or exciting cause only 'an occasion.' These terms, however, are entirely relative For example, in the case of harvest, good rain may be viewed as a proximate cause and sowing as a remote cause; or good rain may be regarded as a remote cause and the industry of the husbandman following it as a proximate cause producing the effect. Thus, the Proximate and Remote Causes are really the direct and indirect 'conditions' which bring about an event. Similarly, Aristotle's enumeration of four causes is really an analysis of the different 'conditions' which conspire to produce an effect.*

(2) The scientific stand-point.

(2) From the scientific stand-point, however, it is essential that all the conditions entering into a cause must be explicitly stated to account for an effect. However insignificant or familiar a condition may be, it needs a mention in science in order to an adequate explanation of the phenomenon under investigation. When, for example, we say that 'the last straw breaks the back of the camel', we do not really mean that the last straw alone does so, though it serves no doubt to bring about the effect in the last instance. A proper or

* The four kinds of causes essential to the production of a thing are called by Aristotle formal, material, efficient, and final. The *formal cause* is the scheme or design by which an effect is produced; the *material cause* is the stuff or substance of which a thing is made; the *efficient cause* is the force or agency by which a result is achieved; and the *final cause* is the end or purpose for which it is produced. Thus, in the case of a building, its plan of construction is the formal; brick, mortar, &c., the material; the masons, the efficient; and the building constructed, the final cause. Aristotle, ultimately resolves these four causes into two—material and formal, the latter as operant design being also the efficient and final cause at the same time.

scientific explanation must enumerate all the conditions, such as (1) the weight of the load from the very beginning, (2) the capacity of the camel to bear the burden, (3) the limit of this capacity, (4) the action of gravity, and (5) the added weight of the straw which brings the weight beyond the capacity of the camel. The scientific view is thus comprehensive in character.

(3) From the stand-point of conservation of energy we should not only enumerate all the conditions essential to the production of an effect, but must also point out the quantitative equivalence of the cause and the effect. The doctrine of conservation of energy teaches us that what we regard as an effect is but the cause transformed. So, we must prove this transformation by indicating their quantitative equivalence. We should remember in this connection that the cause or the effect in any particular case may be illustrated in the form of either a moving power or a collocation.

(3) The stand-point of conservation of energy.

Popular

§ 6. **Plurality of Causes.*** The doctrine of plurality of causes implies that one and the same effect may be produced by different causes: it is not necessarily always produced by the same cause. Light, for example, may be produced now by the sun, now by the moon, and now by fire. "It is not true," says Mill, "that one effect must be connected with only one cause, or assemblage of con-

Plurality of causes implies that the same effect may be produced by different causes.

Mill.

* 'Plurality of Causes' should be distinguished from the 'Composition of Causes.' The former holds that the same effect may be produced by different causes, while the latter means a number of factors or conditions each contributing to the production of a certain effect.

...rine ...on a ...ion: it ...its ...sibility to ...ifference

ditions; that each phenomenon can be produced only in one way. There are often several independent modes in which the same phenomenon could have originated. One fact may be the consequent in several invariable sequences; it may follow, with equal uniformity, any one of several antecedents or collections of antecedents. Many causes may produce motion: many causes may produce some kinds of sensation: many causes may produce death. A given effect may really be produced by a certain cause, and yet be perfectly capable of being produced without it." (*Logic*, I, p. 468.) Thus, one and the same effect, it is urged, may be due to any one of several causes. This is what is called by Professor Fowler, the 'vicariousness of causes.'

Fowler.

The doctrine is untenable.

Though the doctrine of plurality of causes is generally accepted, yet its validity may be questioned. The doctrine is really based on a confusion of the different kinds of effect and an oversight of their differences The light produced by the sun is not of the same character as that produced by the moon; nor is the light of the moon of the same kind as that of the fire. Thus, if we take into account the specific character of the effect in any case, it can be due to only one cause, say the sun, the moon, or the fire. The silver light of the moon is never attributed to the sun, nor is the golden brilliance of the solar ray attributed to the moon. Similarly, though death, generally viewed, may be regarded as the outcome of any one of several causes, yet specifically considered it can be

Illustrations.

due to only one cause. Thus, death from plague is not of the same character as death from fever or poisoning, or from a disease of the lungs or the bowels. Nay, deaths due to different *kinds* of fever (such as black fever, yellow fever, malarious fever), poisoning (such as arsenic, hydrocyanic acid, opium), lung complaints (such as consumption, pneumonia, bronchitis), or bowel complaints (such as cholera, dysentery, and diarrhœa) are not quite of the same character. "Had we been equally exhaustive," observes Venn, "in our enumeration of the constituent elements in the aggregate effect as we were in those of the cause, no such plurality would have been possible. The inclusion of every fresh element among the consequents excludes some of the alternative possibilities of causation, and the inclusion of all would rigidly confine us to one only." (*Empirical Logic*, p. 62.)

The doctrine of 'plurality of causes' as held by Mill is inconsistent with his own view of treating cause as the invariable and unconditional antecedent of a phenomenon. For, the idea of invariable and unconditional sequence carries with it the necessary implication of such a sequence being independent of any other thing, *i.e.*, the implication of a particular set of conditions being able to produce a particular effect or a certain effect being due to none but a particular agency.

The doctrine is based on a confusion: it owes its plausibility to a difference

In fact, the doctrine of plurality of causes is due to our different estimates of the cause and the effect. (1) If we take the cause as generically or vaguely as we take the effect, then we may say that the

in the estimate of the cause and the effect.

same effect is always due to the same cause. Death, for example, is due in every case to the failure of the vital functions. (2) If, again, we be as much precise in our estimate of the effect as we are in our estimate of the cause, then, also, as shown above, the same effect is always produced by the same cause. "We say, for instance,"

Venn's testimony.

writes Venn, "that death may be brought about in a variety of different ways, and we call all these ways 'causes,' and thence deduce the doctrine of plurality of causes. It may be produced by suicide, in any particular case; by disease, and that of various different kinds; by murder; and so forth. But all these alternative suppositions are only rendered possible, because the 'death' is a single element in the sense above described, that is, it has been abstracted from a number of other characterizing circumstances. Had we introduced these other elements or characterizing circumstances, only one of these causes would have been left possible. The condition of the organs would have precluded such and such a form of disease; the position of the body and the nature of the wounds would have precluded the alternative of suicide; and so on with each alternative in turn. So clearly is all this recognized whenever it becomes important to take it into consideration, that the whole procedure in a trial for murder, or in any coroner's court, rests upon the assumption that if we are particular enough in our assignment of the effect there is no possibility left open for any plurality of causes." (*Ibid.*) So far, therefore, as the real constitution

of the world is concerned, there is no justification for the doctrine of plurality of causes, advocated with so much force by Mill and Bain. (*Vide* Bain's *Logic*, II, p. 77.) Generalization of the cause or specialization of the effect brings out that this doctrine is really due to confusion.

The doctrine, however, is consistent with the common view of causation, which ordinarily underlies an inductive inquiry into causes.

It may, however, be said in favour of this doctrine that it is in accordance with the common and popular estimates of causes and effects, with which primarily the logical inquiry is concerned. The perfect equivalence of the cause and the effect and the ultimate unity of the universe are known only to the Supreme Mind, to whom, as we have seen, all inferences are superfluous. (*Vide* Chap. I, § 4 and Chap. IX, § 2.) If, however, all things are not clear to our finite intelligence, we are constrained to have recourse to inference in order to render intelligible what otherwise is obscure and perplexing to us. Hence, when an effect is produced, we try to discover its cause; and an imperfect estimate of it is evidently more satisfactory than a wrong estimate or total ignorance. It is nearer truth, for example, to attribute a burning sensation, in the case of being scorched, to fire than to air or water; and it is surely some relief to be able to discover a cause than merely to gape and be perplexed at a consequence. Hence the inductive canons are intended to exclude wrong conclusions regarding causes or effects; but their end is not to establish that perfect equivalence between cause and effect, which is the consummation of all knowledge and

consonant only with omniscience. In fact, there would be little room for the employment of inductive canons when a knowledge of exact equivalence between cause and effect is reached. Inductive investigation, like every other form of inference, is consistent only with the stage of finite and imperfect intelligence. Perfect knowledge would give rise to intuition all round. (*Vide* Chap. I, § 4 and *Elements of Morals*, Chap. XII, § 3.)

Knowledge of the perfect equivalence between cause and effect would render inductive investigation superfluous.

§ 7. **Conjunction of Causes and Intermixture of Effects.** Owing to the unity, continuity, and complexity of Nature, we seldom find a cause or an effect alone: as a matter of fact, several causes and effects often go together and render it more or less difficult to determine the relation of cause and effect between two phenomena. Even such a simple experience as the sound of a gun or the smell of a flower is usually accompanied by other experiences caused, if not by the same object, at least by other objects affecting other senses. Thus, a flash of light, taste of a lemon, or contact of a gentle breeze may co-exist with such experiences of sound and smell. And, however easy it may seem in our childhood or youth to distinguish the causal connections in such complex cases, the baby has to disentangle the threads by natural intelligence aided by variation of experience, before attributing an effect to its proper cause. We have also seen how what seems to be a single cause is really an aggregate of conditions, determining an effect by their co-pres-

Causes and effects are usually found in complex aggregates; and the inductive problem is to connect a cause with its own effect.

ence; and these conditions in their turn are determined by others, either proximate or remote. Hence we see that the web of our experience is really a very complex fabric, the parts of which are often mixed together and can be distinguished only with great difficulty.

Concurrent causes or effects may either (*i*) remain distinct or (*ii*) blend in an indistinguishable mass, giving rise to homogeneous or heteropathic intermixture of effects. The inductive method is practically limited to the former case (*i*), while the deductive is usually applicable to the latter (ii).

The causes or effects which thus go together may either (i) remain distinct or (ii) amalgamate in a complex whole. The rules or canons of induction are specially applicable to the former case (i), their function being to discover causal connections between certain antecedents and consequents by the elimination of others. When (ii) causes or effects unite in a complex whole, the effect may be either (*a*) *homogeneous* with the conspiring causes or (*b*) *different* in kind from them. (*Vide* Chap. XVII, § 5.) In both the cases we have *intermixture of effects*, the result being in the one case (*a*) compound, while in the other (*b*) heteropathic. (*a*) A *compound effect*, produced by *composition of causes*, is illustrated when two or more forces acting together give rise to a resultant (*e.g.*, when two locomotives double the speed of a train, or when an object pulled in two different directions moves along the diagonal).* This is also known as 'homogeneous intermixture of effects', as in it the

* *The composition of causes* may be distinguished from the *conditions of a cause* by the fact that in the former (*e.g.*, in the case of a tug-of-war or a river fed by two tributaries) the constituent factors would separately produce effects of the same kind, though quantitatively greater or less than the compound effect, while in the latter (*e.g.*, in the case of the explosion of gun-powder) the constituent factors will separately produce no effect of the kind which is the outcome of their joint operation.

compound or total effect is of the *same kind* as its individual causes. As it is not easily amenable to inductive canons, it generally requires the employment of the Deductive Method for a discovery of the true causes; and the more numerous the factors which bring about an effect, the greater the difficulty of using the inductive method, owing to the comparatively small share of each cause in producing the effect, and hence the greater the necessity of applying the Deductive Method. (*Vide* Chap. XX.) (*b*) A *heteropathic effect*, produced by a *combination of causes*, is illustrated when an unlike result is produced by co-operant causes, such as we find in the production of water by a chemical combination of oxygen and hydrogen. The application of the inductive canons is possible in this case, though the employment of the deductive method is more efficacious here also. The deductive method proceeds by supposing the different causes from which the complex effect is synthetically deduced by reference to the laws of their operation; and this is a more practicable course in the case of the intermixture of effects (specially when it is homogeneous with the causes) than the employment of the inductive canons, which can scarcely work by elimination here, since the different parts of the effect inextricably blend in a homogeneous whole.

Effects may sometimes be present in the form of tendency or tension.

It may be mentioned in this connection that sometimes different effects may be so opposed that they are apparently destroyed, as in the case of a bent bow or an evenly balanced tug-of-war. Really, however, the effects in such cases are not extin-

guished but held in reserve in the form of tendency or tension, as is evident from the manifestation of an effect when the string is cut in the one case or a member is withdrawn from either side in the other. Neutralization should not be construed as annihilation. A is the cause of B — B is the cause of A

taken in (abstract sense

§ 8. Mutuality of Cause and Effect. We have seen that no sharp demarcation line can be drawn between causes and effects, they being but different aspects of one and the same energy. (*Vide* § 4.) The application of a spark to gunpowder, for example, is the cause of its explosion; but this explosion again is the cause of the sensation of sound, which in its turn may be the cause of startling or of vigilance. Thus, we can never say that anything is absolutely a cause or an effect. What may be a cause with regard to one may be an effect with regard to another. Not only so, we find that often causes and effects re-act on each other. Industry, for example, promotes thrift, and thrift in its turn encourages industry; sympathy secures co-operation, and co-operation again fosters sympathy. Owing to the continuity and activity of all natural processes, such action and re-action are illustrated in all the departments of nature and mind.

Cause and effect are entirely relative, they being but different aspects of the same energy.

The course of Nature is continuous, in which an event may be viewed as an effect in relation to a prior phenomenon, while as a cause in reference to a posterior.

Cause and effect often re-act on each other.

Sir G. C. Lewis well illustrates this reciprocity in the case of political causation. "It happens sometimes," he writes, "that when a relation of causation is established between two facts, it is hard to decide which, in the given case, is the cause and which the effect, because they act and

Testimony of Sir G. C. Lewis.

re-act upon each other, each phenomenon being in turn cause and effect. Illustrations. Thus, habits of industry may produce wealth; while the acquisition of wealth may promote industry: again, habits of study may sharpen the understanding, and the increased acuteness of the understanding may afterwards increase the appetite for study. So an excess of population may, by impoverishing the labouring classes, be the cause of their living in bad dwellings; and, again, bad dwellings, by deteriorating the moral habits of the poor, may stimulate population. The general intelligence and good sense of a people may promote its good government, and the goodness of the government may, in its turn, increase the intelligence of the people, and contribute to the formation of sound opinions among them. Drunkenness is in general the consequence of a low degree of intelligence, as may be observed both among savages and in civilized countries. But, in return, a habit of drunkenness prevents the cultivation of the intellect, and strengthens the cause out of which it grows. As Plato remarks, education improves nature, and nature facilitates education. National character, again, is both effect and cause: it re-acts on the circumstances from which it arises. The national peculiarities of a people, its race, physical structure, climate, territory, etc., form originally a certain character, which tends to create certain institutions, political and domestic, in harmony with that character. These institutions strengthen, perpetuate, and reproduce the character out of which

they grow, and so on in succession, each new effect becoming, in its turn, a new cause. Thus, a brave, energetic, restless nation, exposed to attack from neighbours, organizes military institutions: these institutions promote and maintain a warlike spirit: this warlike spirit, again, assists the development of the military organization, and it is further promoted by territorial conquests and success in war, which may be its result—each successive effect thus adding to the cause out of which it sprung". (*On Methods of Observation and Reasoning in Politics*, Vol. I, p. 375.)

Observation and Experiment supply materials for inductive generalizations.

§ 9. **Observation and Experiment.** Observation and Experiment constitute, as we have seen, the material grounds of induction; for every inductive generalization materials must be presented to the mind, which constitute the subject-matter of such a generalization. But merely the presentation of materials is not adequate; there must also be an observing mind. And, in order to arrive at a correct generalization, we must observe with care, attention, and patience. Mere random perception of events or objects can never be conducive to inductive inquiry. If we notice any and every object which comes before the mind, then there will be no system and no possibility of sound generalization. Hence simple perception, or knowledge of objects or phenomena as presented to the mind, is not favourable to inductive research. In order to it, we must regulate our perception in the required direction: we should know what we are about, withdrawing our attention

from what is irrelevant and directing our attention to what is connected with our inquiry. Thus, observation should be distinguished from mere perception: we may describe *observation* as *regulated perception*. And it is this regulated perception or observation which is of special importance in inductive inquiry.

Observation is regulated perception.

According to some, *e.g.*, Stock, "Observation is passive experience" and "Experiment is active experience", on the ground that in the former the mind is passive, as it merely receives impressions about the phenomena under investigation, whereas it is active in the latter. But such a distinction is based on an erroneous theory of knowledge, *viz.*, mental passivity. In fact mental activity is involved in both in the form of interpretation of sense-impressions. We must construe observation, however, in a broad sense, so as to include experiment. Indeed, observation and experiment are not distinct in kind. They are but forms of regulated and prolonged perception, conducted with great care and attention and having some definite end in view. In both the cases, facts are presented to the mind and the mind takes notice of such facts. Though, however, the mind cognizes facts in both the cases, yet there is a difference in the procedure by which facts are brought before the mind. As Bain puts it, "Observation is *finding* a fact, Experiment is *making* one": in the one case, we merely notice a fact which in the course of nature comes before the mind; while, in the other, we so mould the circumstances as to give rise

In a wide sense Observation includes Experiment.

Observation and Experiment differ in procedure.

to the fact or phenomenon which we wish to observe. In observation it is not always possible for us to simplify the conditions and have a control over them. But in experiment we attempt the production of a similar phenomenon under conditions which are simpler and variable at our will.* Electricity, for example, in the form of lightning is observed in the case of atmospheric discharge when it happens; but it may be produced at pleasure in the laboratory by experiment, *i.e.*, by an arrangement of circumstances brought about by voluntary agency. Bacon likened Experiment to cross-examination of witnesses, since by means of it we so interrogate Nature as to elicit definite answers from her. Thus, Observation and Experiment are the channels of information which supply materials for inductive generalizations.

Relative advantages of Observation and Experiment: **(I) The advantages of Experiment.**

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Though Observation and Experiment are thus essentially the same, yet the difference in procedure in the two cases brings out an important difference in their relative advantages. (I) The advantages of Experiment ~~over Observation are:~~

(1) In the case of Experiment we can multiply the instances as often as we like, while in the case of Observation we must depend upon the bounty of Nature for a suitable opportunity.

(2) In the case of Experiment we can produce an effect or phenomenon under definite and known

* Sometimes we find in nature certain conditions happening by themselves without our agency and offering an opportunity for the special study of a phenomenon. These are called by some *Natural Experiments*, *e.g.*, a lunar eclipse which is so favourable to proving the round shape of the earth.

accurate and Decisive

circumstances, while in the case of Observation there may be innumerable unknown and unsuspected agencies which modify the phenomenon under observation.

(3) In the case of Experiment we may vary the circumstances as we like, while in the case of Observation we must depend entirely on the special combination which Nature may be pleased to present before us.

(4) In the case of Experiment we can observe with greater care, while in the case of Observation we are taken by surprise, as it were, and thus have not an opportunity to observe with the same degree of caution and precision.

In the case of Experiment we can not only adjust our attention to the circumstances bringing about an effect, but we can also observe with a better care the issue or result. Attention, in the case of Experiment, is pre-adjusted; and hence it enables us to notice with greater care what otherwise might have been overlooked or but cursorily examined. "To experiment," says Fowler "is, not only to observe, but also to place the phenomenon under peculiarly favourable circumstances, as a preliminary to observation." (*Inductive Logic*, p. 34.)

Illustrations. The truth of the above remarks is illustrated in the following examples mentioned by Minto in his *Logic*:—

"The air-pump was invented shortly before the foundation of the Royal Society, and its members made many experiments with this new means of

isolating an agent and thus discovering its potentialities. For example, live animals were put into the receiver, and the air exhausted, with the result that they quickly died. The absence of the air being the sole difference, it was thus proved to be indispensable to life. But air is a composite agent, and when means were contrived of separating its components, the effects of oxygen alone and of carbonic acid alone were experimentally determined.

"A good example of the difficulty of excluding agencies other than those we are observing, of making sure that none such intrude, is found in the experiments that have been made in connexion with spontaneous generation. The question to be decided is whether life ever comes into existence without the antecedent presence of living germs. And the method of determining this is to exclude all germs rigorously from a compound of inorganic matter, and observe whether life ever appears If we could make sure in any one case that no germs were antecedently present, we should have proved that in that case at least life was spontaneously generated.

"The difficulty here arises from the subtlety of the agent under observation. The notion that maggots are spontaneously generated in putrid meat, was comparatively easy to explode. It was found that when flies were excluded by fine wire-gauze, the maggots did not appear. But in the case of microscopic organisms proof is not so easy. The germs are invisible, and it is difficult to make

certain of their exclusion. A French experimenter, Pouchet, thought he had obtained indubitable cases of spontaneous generation. He took infusions of vegetable matter, boiled them to a pitch sufficient to destroy all germs of life, and hermetically sealed up the liquid in glass flasks. After an interval, micro-organisms appeared. Doubts as to the conclusion that they had been spontaneously generated turned upon two questions: whether all germs in the liquid had been destroyed by the preliminary boiling, and whether germs could have found access in the course of the interval before life appeared. At a certain stage in Pouchet's process he had occasion to dip the mouths of the flasks in mercury. It occurred to Pasteur in repeating the experiments that germs might have found their way in from the atmospheric dust on the surface of this mercury. That this was so was rendered probable by his finding that when he carefully cleansed the surface of the mercury no life appeared afterwards in his flasks." (Pp. 314-315).

Though Experiment possesses certain advantages over Observation, yet it can scarcely be denied that Observation also possesses certain advantages over Experiment. (II) The advantages of Observation over Experiment are:—

(II) The advantages of Observation.

(1) There are facts or phenomena which are beyond our control, as well as agencies too dangerous to be experimented upon. We cannot, for example, have recourse to experiment with regard

e.g., to heavenly bodies, which are altogether beyond

our control, and it will be too dangerous to have recourse to experiments involving serious injury to or destruction of individuals or society. In such cases we must be content with the observation of facts or phenomena as presented to us.

(2) In the case of Observation we may direct our attention from the effect to the cause or from the cause to the effect. Noticing instances of invariable sequence, we may observe the effect as well as the cause. But, in the case of Experiment, we proceed from the cause to the effect: the cause being given or surmised, we try by experiment to find out its effect. We can make the cause produce its effect; but we can never make the effect bring back or reproduce its cause. All that we can do in the latter case is to suppose a cause by reference to past experience and then to make it produce its effect. Experiment thus leads us from cause—real or supposed—to effect, while Observation in either direction.

In experiment.

Experimental sciences are more progressive than those resting on observation alone.

It is clear from the preceding account that, of the inductive sciences, those which can employ both observation and experiment are generally more certain and advanced than those which rest on observation alone. If we can actively manipulate materials so as to produce certain results at our will, we have means of verification under our control; and thus we can often establish truths more conclusively under such circumstances than when we are left merely at the mercy of Nature. This is clearly brought out by the fact that while the physical sciences made but little progress in

ancient and mediæval times owing to the total neglect of experiment by the Greek and mediæval scholars, these very sciences have made rapid strides in modern times owing to the extensive use of experiments in them now-a-days. And thus these sciences are ranked as prominent among the experimental sciences as distinguished from those in which, from the very nature of the subject-matter, we are constrained to have recourse to observation alone (*e.g.*, geology and astronomy).

Observation and Experiment are often vitiated by inference.

Before closing this section we must guard against an error which often vitiates our observation and experiment. We not infrequently mix up what is actually presented with what is but imagined or supposed. Thus, on seeing a cord we think it to be a snake or on noticing a sudden flash of light we take it to be lightning. In such cases perception is confounded with inference, leading to a wrong reading of facts and so to false surmises and incorrect conclusions. (*Vide* Chap. XXX, § 8). We, should, therefore, be very careful in our observations and experiments, so as to take them at their proper worth and not to import into them what is suggested from without, in order that our inferences from them may be valid.

Observation, Explanation, and Classification are interconnected.

§ **10. Observation and Explanation.** We should remember in this connection that Observation is not an aimless act but is employed for some end in view. We *observe* a natural phenomenon (say, an eclipse of the sun or the moon) or have recourse to an experiment in the laboratory as illustrating some truth or law. Observations are

actually controlled or regulated by some guiding idea involved in a theory or an effort at explanation; and the facts observed in their turn furnish materials to confirm or refute the theory or hypothesis. Observation and Explanation are thus inter-connected: we never observe in a random way; and the materials which we gather under the spell of explanation in their turn contribute to it. Moreover, observation (including experiment) is a mental act involving comparison and leading to the discovery of relations of similarity and difference; and whenever we bring a phenomenon under one class as distinct from another, we mean that it is governed by certain fixed relations and laws which explain its character and behaviour. We, accordingly, find that Observation, Explanation and Classification usually go hand in hand. (*Vide* Chap. XXVI, § 2.)

§ **11. Ground of Causation.** Opinions differ with regard to the ultimate basis of the Law of Causation. We know that, in order to arrive at an inductive generalization, we must employ certain tests or, as they are called, Inductive Canons. These Inductive Canons in their turn presuppose the Law of Causation, for, without our faith in such a law, the Canons by themselves can never warrant inference from the known to the unknown. Now the question is, What is the ground of this Law of Causation itself, which is taken to be the basis of the Inductive Canons? Two different answers have been given to this question:—

Different views of the origin of our belief in Causation.

(1) The Law of Causation is taken by some

(1) The

Intuitional View.

(*viz.*, by intuitional writers) as inherent in our mental constitution. Our mind has been moulded, as it were, according to this law. Thus, whenever an event takes place in Nature, we are led by this natural law of our understanding to think that it must have a cause. Such a tendency is at once universal and necessary, and the causal principle is self-evident in character.

(2) The Empirical View.

(2) It is urged by the supporters of the empirical school that the Law of Causation, like other laws, is but a generalization from experience. We have observed that every event has a cause; and consequently we infer that the causal law is universal in character. But this position is inconsistent and untenable. As Mansel observes, it is a 'paralogism' to hold that 'the ground of all Induction is itself an Induction'. We have seen that all inductive generalizations ultimately rest on the assumption of the causal principle; and if this principle itself be a generalization from experience, then we move in a never-ending circle.

The empirical view moves in a circle.

It is urged by empiricists that Universal Agreement, which is the basis of Causation, is not the same as the Experimental Method of Agreement.

Mill and Bain, no doubt, defend the empirical position by holding that the ground of the Law of Causation is not exactly the same as the ground of an inductive generalization. The Law of Causation is due to Universal Agreement throughout nature: it is this uncontradicted experience which induces belief in the universal character of the causal principle. And this Universal Agreement should not be confounded with the Method of Agreement as an Inductive Canon, which proceeds by elimination and varying the circumstances. No

elimination or variation of circumstances is implied in the Universal Agreement which, according to the supporters of this view, is the final basis of all ultimate laws.

Causation, however, is not proved by Universal Agreement.

But, it may be replied that experience alone can never be an adequate ground for establishing the Law of Causation. If we appeal to experience, we find that we know the causes of but few events or phenomena, while we are ignorant of causes in innumerable instances. Thus, if experience be the guide, then we should rather be led to hold that events are uncaused than caused. Moreover, experience can scarcely beget such a necessary and universal conviction as we find attending the causal principle. It thus appears that the law of Causation, which is taken to be the formal ground of every inductive inference, is an *a priori* axiom of the understanding.

Experience fails to account for the necessity and universality involved in the causal relation.

Causation is ordinarily regarded as the formal ground of induction; but, properly speaking, Causation and Uniformity of Nature constitute together such a ground.

§ **12. Relation of Causation to Uniformity of Nature.** It is generally believed that the Law of Causation alone is the formal ground of Induction. This is the prevailing opinion; and there is some justification for it when the cause is construed as the invariable antecedent of an effect or phenomenon. But this invariable sequence is connected rather with the Law of Uniformity of Nature than with the Law of Causation itself. The Law of Causation, strictly speaking, may be taken to imply that every event or change ...st have a ground or cause. But though every ...s thus believed by us to be caused, yet it ...not necessarily follow that the same cause

should always produce the same effect. The latter statement implies not merely that an effect must have a cause, but also that Nature is uniform in her procedure, yielding the same result under the same circumstances. Thus, both the Law of Causation and the Law of Uniformity of Nature seem to be the indispensable formal grounds of inductive generalizations.

(a) Without Causality, mere Uniformity can never justify an inductive leap.

(*a*) Without the Law of Causality we may generalize, but we cannot arrive at Induction in the proper sense of the term : relying on the Law of Uniformity of Nature we may generalize relations not known to be due to causal connection. In such cases it would be precarious to proceed from the known to the unknown, in the absence of a known tie or connection between phenomena ; and such instances may at most amount to Immethodical Induction or Induction by Simple Enumeration.

(b) Without Uniformity, mere Causation can never justify a generalization.

(*b*) Without the Law of Uniformity of Nature we can never generalize at all, even though we believe in the Law of Causation : effects may be caused ; but if the same cause be not always followed by the same effect, then evidently there can be no valid ground for generalization or proceeding from the known to the unknown. If, for example, fire now burns and now quenches thirst, and the ground is now solid and now liquid, or, say, rice is now nutritious and now poisonous, then evidently we can never arrive at a correct general proposition.

Mill and Bain take Causation as

Mill and Bain and many other logicians do not include the Law of Uniformity of Nature as a

distinct condition in the formal ground of Induction, because according to them the Law of Causation implies the Law of Uniformity of Nature. The Law of Causation, it is said, is but an instance of the Uniformity of Nature, namely, uniformity with regard to sequence. Hence is it that Mill and Bain contend that, in every inductive generalization, the ultimate major premise must be the Law of Uniformity of Nature, which is taken to be the ground of Causation itself. But, as we have seen, Causation is essentially distinct from Uniformity of Nature. Hence, Causation by itself can never be taken as the formal ground of induction; but *Causation as modified by the Law of Uniformity.*

but an aspect of Uniformity but this is not true.

We may mention here the views of some empiricists (*e.g.*, Mill and Bain) who regard the Law of Uniformity of Nature as but a generalization from experience. Thus, according to them by observing several instances of burning of fire and floating of timber in water we believe that fire and timber will similarly behave in future under similar conditions. This belief, then, in the Uniformity of Nature is derived from the uniform experience of the past. But it may be said, as against this view, that such an attempt of explaining the Law of Uniformity as an empirical generalization is hardly tenable, as such a generalization presupposes Uniformity of Nature. If we did not believe, for instance, that fire would burn and timber would float as they did in the past, we could not have inferred their uniform behaviour in the future on the strength of our previous experience about

The empirical view holding Uniformity as but generalization from experience is also untenable.

them. Thus, the empirical account of the origin of our belief in the Uniformity of Nature is a paralogism, as Mansel says, because it views the belief in Uniformity as a result of generalization and yet presupposes this belief in every generalization.

§ 13. **Exercises.**

1. What are the grounds of Induction? Distinguish between Formal and Material Grounds. Explain and examine the remark—"The Principle of Causation is the formal ground of Induction."

2. Indicate the character and marks of the Law of Causation and the Law of Uniformity of Nature.

3. How is the Principle of Uniformity of Nature related to the Law of Causation in its scientific aspect? Distinguish between the scientific and the popular view of Causation.

4. Explain the meanings of Energy and Conservation of Energy; and show the bearing of the Theory of Conservation on the nature of Causation.

5. Distinguish between (*a*) Causes and Conditions and (*b*) Proximate and Remote Causes. Elucidate—"The cause is the sum total of the conditions, positive and negative, taken together."

A man is crossing the river in a small boat; a sudden squall of wind comes on; the boat founders, and the man is drowned: what do you consider to be the cause and the conditions of his death?

A balloonist, unable to make a successful parachute descent, falls headlong and dies. Determine clearly the cause and conditions of his death.

6. What do you understand by Plurality of Causes? Is the doctrine strictly tenable?

7. Distinguish between (1) joint and complex effects and (2) homogeneous intermixture and heteropathic intermixture of effects. Are the effects in the case of an evenly balanced tug-of-war destroyed?

8. What do you understand by the Mutuality of Cause and Effect? Illustrate your remarks by examples.

9. Distinguish between Observation and Experiment, and indicate their relative advantages in inductive investigation. Elucidate—"Observations and experiments are the material grounds of Induction."

10. Show by a careful estimate of Observation and Experiment that the difference between them is 'not of kind but of degree.' What sciences depend mainly on Observation and why? What sciences depend mainly on Experiment and why? Do Observation and Experiment alone justify an inductive generalization?

11. What is the ultimate ground of our belief in Causation? Discuss the different views on the subject.

12. Distinguish between the Conditions of a Cause and the Composition of Causes. Examine in detail the statement that a cause is the *immediate, invariable, and unconditional antecedent* of an effect.

CHAPTER XVIII.

THE EXPERIMENTAL METHODS.

§ 1. **Experimental Methods as Deductions from Causation.** We have seen that the difference between scientific and unscientific induction lies in the systematic employment of certain logical tests or methods which secure a degree of certainty in the one case, not attainable in the other. Thus, the Experimental Methods, or Canons of Induction, as they have been called, serve the same purpose in Induction as do the rules of the syllogism in deductive reasoning. And, as the syllogistic rules are mere deductions from the principles of consistency, so the Experimental Methods are really deductions from the Law of Causation as it is construed in science, *i.e.*, as modified by the Law of Uniformity of Nature. (*Vide* Chap. XVII, § 4 and § 12.)

As the syllogistic rules are deductions from the principles of consistency, so the inductive canons are deductions from the law of causation as conceived in science.

Our aim in every inductive inquiry is to generalize a relation from the observation of several individual instances. And there is a legitimate ground of such generalization when we detect causal connection between the phenomena about which we wish to generalize. (*Vide* Chap. XVI, § 2.) To detect the causal connection is, however, not always an easy task, as most phenomena presented to us are of a complex character. (*Vide* Chap. XVI, § 1.) Hence the necessity of elimination or exclusion of those circumstances which,

The Inductive Canons aim at discovering the causal connection by

though accompanying the phenomena under investigation, are not vitally connected with them. (*Vide* Chap. XVI, § 4.) The different Inductive Canons or Methods thus aim at excluding such accidental circumstances and discovering the factors that are really connected by causation. "The gist of all the Methods," observes Dr. Venn, "by which we are enabled to isolate the cause, and to determine over what limits it may safely be inferred, is one of analysis and exclusion." (*Empirical Logic*, p. 352.) And herein lies the great merit of Mill's System as distinguished from the Baconian Method. Bacon simply pointed out the necessity of observation that we may not be misled by our fancies but arrive at correct generalizations in harmony with facts. We should remember, however, that mere observation of facts is but a precarious guide without the employment of systematic procedure to explain them by the rejection of the inert factors and the selection of those that are potent. While Bacon was content with emphasizing the necessity of a mere careful study of Nature (facts). Mill aimed at an exact formulation of general Methods of Induction which might be equally applicable to the material and moral sciences. The conditions of proof being the same in all cases, he tried to formulate Rules or Canons to satisfy them. As observation is not the sole ground of Induction, we must, according to him, methodically generalize, *i.e.*, we should try to explain facts by reference to laws gathered by well-regulated inductive investigation.

variation of circumstances and elimination.

Dr. Venn.

Features of Causation from which the Canons follow:

The several Inductive Canons or Methods, which we shall study in this chapter, follow directly from the Law of Causation. We have read that the Law of Causation, as conceived in Logic, implies that the cause is the invariable, unconditional, and immediate antecedent of what follows (*viz.*, the effect). An explanation of causal relation, therefore, depends on two sets of conditions: (1) that a certain cause always produces a certain effect or that whenever X (cause) is Y (effect) is, or whenever Y is, X is; and (2) that a certain effect does not happen in the absence of a particular cause, or that whenever X is not, Y is not. The former, that is (1), is usually called a *Positive Instance*, while the latter, that is (2), a *Negative Instance*. Negative Instances, however, should be distinguished from *Exceptions*. Exceptions disprove causal relation, while Negative Instances prove it. We can now deduce the following traits or features which necessarily follow from the nature of a cause and which enable us to decipher the causal relation in any case by the extrusion of accidental circumstances:—

(i) An antecedent which can be excluded without prejudice to the effect is no part of the cause.

(*i*) No variable or accidental circumstance is the cause or part of the cause of a phenomenon.* Thus, when the relation between an antecedent and a consequent is a variable one, *i.e.*, when the one exists without the other, the relation can

* This is evidently the contrapositive of the definition of 'cause.' It may be proved thus:—A cause is the invariable and unconditional antecedent of an event: therefore, no cause is a variable or accidental antecedent of an event (*obverse*): hence, no variable or accidental antecedent of an event is its cause (*contrapositive*).

never be a causal one. We, accordingly, conclude that whatever antecedent can be left out, without prejudice to the effect or whatever antecedent can be present, without the effect being present, can not be its cause or part of its cause. If, therefore, there be circumstances or antecedent phenomena which may be removed without affecting the effect in the least, then these plainly have nothing to do with the cause. If, for example, my lecture is not affected in any way by the absence of the register or the table before me, then evidently these are not causally connected with it. From this it also follows that if the introduction of a phenomenon does not influence an effect in the least, then such a phenomenon cannot be regarded as its cause or part of its cause. If, for instance, my lecture is not at all affected by the entrance of a bearer into the room, then there can be no causal connection between the two phenomena. The very fact that the effect existed when the phenomenon was absent serves as a case of natural elimination, bringing out the non-existence of causal nexus between the two. We shall see that this trait of causation is prominently illustrated in the *Canon* or *Method of Agreement*. (*Vide* § 4.)

(This is the basis of the Canon of Agreement.)

(*ii*) The invariable and unconditional antecedent of a phenomenon is its cause.* From this it is clear that whatever antecedent cannot be left out, without prejudice to the effect, must be its cause

(ii) An antecedent which cannot be excluded without

* This is the simple converse of the definition of 'cause.' A definition, we know, as the explanation of a term, admits of *simple* conversion. (*Vide* Chap. X, § 2.)

affecting the effect is a part of the cause.

or part of its cause. The relation between cause and effect being an inseparable one, we naturally conclude that what cannot possibly be separated from an effect, without impairing or annihilating it, is its cause or a necessary part of its cause. If, for example, light can never be removed without obscuring my vision, then I am led to infer that light is the cause or a necessary condition of vision. We should, however, remember here that the unconditional character of the causal relation is essential to justify an inference that what necessarily precedes is the cause of what necessarily follows, as otherwise the necessary connection may be due to some remote cause of which the antecedent and the consequent are but co-effects (*e.g*, the succession of day and night). Thus, in order to be sure that an invariable antecedent is the cause, we must satisfy ourselves that nothing else goes with it (either in an overt or in a covert form) which might account for the effect. If the removal of a certain antecedent in any case involves also the removal of another potent factor associated with it, then we cannot rightly infer that the antecedent is the cause of the consequent, even if the consequent disappears with the disappearance of the antecedent, for the absence of both may be due to the removal of the unsuspected potent factor. Hence, *other circumstances remaining unaltered*, if we find that an antecedent cannot be excluded without the exclusion of a consequent, then we may take the one to be the cause or a necessary condition of the other. This trait of causation underlies,

(This is the basis of the

as we shall see, the employment of the *Canon* or *Method of Difference*.

Canon of Difference.)

(*iii*) The effect being but the cause transformed, there is equivalence between them, which is emphasized in the quantitative aspect of causation. Thus, any increase or decrease of the one is invariably followed by a corresponding increase or decrease of the other. If, for example, one horse draws a carriage at the rate of 3 miles an hour, two horses of the same strength would draw it at the rate of 6 miles an hour. When, therefore, an antecedent and a consequent vary in numerical concomitance, either directly or inversely, we conclude that they are causally connected. This trait of causation underlies the *Canon* or *Method of Concomitant Variations*. When, accordingly, we find that the rise or fall of an antecedent, which cannot altogether be eliminated, is accompanied by the rise or fall of a certain consequent, we naturally conclude that the antecedent and the consequent thus varying in numerical concomitance are causally connected.

(iii) Antecedents and consequents varying in numerical correspondence are causally connected.

(This is the basis of the Canon of Concomitant Variations.)

As the Joint Method (called also by Mill 'The Indirect Method of Difference,' by Bain 'The Method of Double Agreement,' and by Fowler 'The Double Method of Agreement') and the Method of Residues presuppose, as we shall see, the other Methods or Canons, all these Canons or Methods are thus seen to be ultimately based on the Law of Causation. Thus, Induction really aims at drawing a universal conclusion consistently with the data furnished by experience and the light

As the Canon of the Joint Method and that of Residues presuppose the other Canons, all the Canons are found to depend on Causation

supplied by the Law of Causation. "In fact," says Mr. Read, "Inductive Logic may be considered as having a purely formal character. It consists, first, in a statement of the Law of Cause and Effect; secondly, in certain immediate inferences from this Law, expanded into the Canons; thirdly, in the syllogistic application of the Canons to special propositions of causation by means of minor premises, showing that certain instances satisfy the Canons." (*Logic*, p. 225.) Bain, likewise, observes that the several Inductive Methods are really Deductive, as they follow directly from the Law of Causation; they are called Inductive only "by courtesy."

Testimonies of Read

and Bain.

§ 2. **Enumeration and Analysis of Instances.** From the preceding remarks it is clear that the aim of Induction is to arrive at a universal proposition by establishing causal connection between two facts or phenomena. Induction thus presupposes that the world is a system or cosmos and not merely a disconnected aggregate of things or a chaos. We believe that all things have definite natures of their own by reason of which they behave in certain uniform ways towards one another. This evidently implies that all things of the world constitute an interconnected whole, so that a change in one gives rise to a change in another; and this reciprocity is governed by law, which defines what we call the nature of a thing. The end of Induction, then, is to discover this nature or law governing a relation in any case. We thus say 'Heat expands bodies'

Induction aims at discovering a causal connection and thereby establishing a universal proposition.

'Man is mortal,' 'Matter gravitates,' *etc.* (*Vide* Chap. XV, § 3 and Chap. XXIII, § 2.)

Instances are observed with a view to discover the causal connection.

As, however, the nature or law is embodied in things, we must study them with care in order to discover it aright. Hence the necessity of observation and experiment, which bring before our mind appropriate instances calculated to reveal the inner law of their nature. The instances or facts being, however, generally complex, we must have recourse to analysis and elimination in order to exclude the accidental adjuncts and thereby to discover the essential nature underlying them.

Enumeration of instances is thus an aid to induction.

Mere multiplication of instances, therefore, does not mean much, unless we use the instances as means of discovering their inner nature revealing laws of connection. Perfect Induction or Induction by Complete or Simple Enumeration, therefore, falls short of the requirements of true Induction. (*Vide* Chap. XVI, § 7 and Chap. XXII, § 1). Enumeration is but preparatory to Analysis. All that Enumeration of Instances can do is to create a presumption in favour of or against the presence of a law, according as they are uniform or conflicting in character; and, in the case of preponderance of evidence, we generally rely on statistics to determine a law, which is disguised by the influence of hostile elements. (*Vide* Chap. XXI, § 3 and § 4). Usually, however, Induction by Simple Enumeration is the starting-point of Scientific Induction: several similar instances first suggest a law connecting them; and then we try to verify it by further observation, analysis, and comparison.

Adequate analysis of even a single case may justify an inductive generalization.

From this we can easily see that, for the purpose of true inductive generalization, adequate analysis and examination of even a single case may at times suffice; such, for example, as we gather from the dissection of an animal or the chemical analysis of a compound, unhampered by distracting circumstances. (*Vide* Chap. XXII, § 6.) A true inductive generalization, due to the nature or necessary connection of things, may appropriately be expressed in the form of a hypothetical proposition implying a relation of dependence between an antecedent and a consequent. Thus, 'All men are mortal' may be expressed as 'If humanity is, mortality is.' (*Vide* Chap. VII, § 5.)

A true induction may appropriately be expressed in the form of a hypothetical proposition.

Logic as a science of proof tests hypotheses about causation.

§ 3. Inductive Methods as Weapons of Elimination. We have already read that Proof, and not Discovery, is the end of Logic. (*Vide* Chap. I, § 11.) Logic does not teach us how to discover the causal connection in any particular instance. Such a discovery must be suggested to our mind by imaginative insight. We must first frame some hypothesis to explain the phenomenon under investigation, before a discovery can be made of its appropriate cause.

A hypothesis, as a guiding conception, enables us to select appropriate instances by reference to which we can test its validity. As the instances, however, are more or less complex in character, we have to analyse them and eliminate the accidental features before we are able to find out the material factors which are likely to throw light on the hypothesis. The Rules by which we test these in-

stances, with a view to the exclusion of the inert accompaniments and the selection of potent factors, are known as the Inductive Canons or Methods. These Canons or Inductive tests are five in number, *viz.*, (I) the Canon of the Method of Agreement, (II) the Canon of the Joint Method, (III) the Canon of the Method of Difference, (IV) the Canon of the Method of Concomitant Variations and (V) the Canon of the Method of Residues. These Canons are chiefly applicable to cases where the causes and effects remain distinct, instead of blending in a homogeneous whole, when, as we have said, the deductive method is specially applicable. (*Vide* Chap. XVII, § 7.)

Five tests or Canons.

The Canons are specially useful when the causes and effects remain distinct.

The Inductive Methods have at times been described as Weapons of Elimination; but this description is only partially true. No doubt, we try, by the employment of the Canons or Methods, to exclude the accidental circumstances, which obscure the potent factors and thus stand in the way of the discovery of the causal connection. But we should remember that the function of the Inductive Methods is not mere negative exclusion but also positive selection: if they are employed for the elimination of inert adjuncts, it is only because such elimination clears the ground and so renders prominent the factors which are related as cause and effect. Thus, the aim of the Inductive Methods is always positive discovery, though this is attained by negative exclusion: the irrelevant are thrust out in order that the relevant may be seen. To single out the essential factors which are

The Inductive Methods employ elimination with a view to discovery.

really operative, the Inductive Methods have recourse to the elimination or rejection of what are variable and inert.

Thus, each Inductive Method has both a positive and a negative function, following immediately from the traits of causation explained above.

From the above remarks it is clear that each of the Inductive Methods or Canons has both a positive and a negative aspect; and these two aspects readily follow from the first two traits or features of causation indicated in section before the last. The first trait has the negative function of elimination; while the second, the positive function of finding out the causal relation. The Method of Concomitant Variations follows evidently from the third trait or feature. With these preliminary remarks let us now proceed to explain the five Inductive Canons one by one.

Enunciation of the Canon.

§ 4. (1) The Canon of Agreement. *If two or more instances of a phenomenon under investigation have only one other circumstance (antecedent or consequent) in common, that circumstance is the cause or the effect of the phenomenon, or connected with it by causation.*

This Canon implies that when we find a phenomenon *uniformly* preceded or followed by another in two or more cases, then we are led to conclude that this other phenomenon is the cause or the effect, according as it precedes or follows the phenomenon under investigation, or the two phenomena are co-effects of some other cause, so that the one can never be found without the other. Whether we are right or wrong in our supposition can, of course, only be proved by more extended

observation or by the employment of the other Canons. As explained in Chap. XVI, § 4, elimination by variation of circumstances is prominently illustrated in this Method.

Illustrations.

If, for example, *A* B C be followed by *a* b c, *A* D E by *a* f g, and *A* R S by *a* m n, then we naturally suspect that the common factor in the antecedents (*viz.*, *A*) is the cause or a necessary part of the cause of the common factor in the consequents (*viz.*, *a*). To take a concrete example: if an individual repeatedly suffers from rheumatic pains when the weather is cloudy, then he naturally suspects that the cloudy weather is the cause or a necessary condition of his rheumatism.

This Canon is employed in Observation.

The Canon of Agreement is essentially a Canon of the Method of Observation. We observe numerous instances of correspondence between antecedents and consequents, from which we naturally conclude that there is some causal connection between the common factors present in them. We find, for example, a remarkable coincidence between the scarlet colour and the absence of fragrance. The following account in this connection is instructive:—"Among all the colours that blooms assume, none are less associated with fragrance than scarlet. We cannot at present recollect a bright scarlet blossom that is sweet-scented—yet no other colour among flowers is more admired and sought after. Scarlet prevails among Balsamina, Euphorbia, Pelargonium, Poppy, Salvia, Bouvardia, and Verbena, yet none of the scarlets are of sweet perfumes. Some of the light-coloured

Illustrations.

Balsams and Verbenas are sweet-scented, but none of the scarlets are. The common Sage, with blue blooms, is odoriferous both in flower and foliage; but the scarlet-Salvias are devoid of smell. None of the sweet-scented-leaved Pelargoniums have scarlet blooms, and none of the scarlet bloomers have sweet scent of leaves nor of blooms. Some of the white-margined Poppies have pleasant odours; but the British scarlets are not sweet-scented. The British white-blooming Hawthorn is of the most delightful fragrance; the scarlet-flowering has no smell. Some of the Honeysuckles are sweetly perfumed, but the Scarlet Trumpet is scentless." (Elder, *American Gardener's Monthly*.)

Defects of the Canon:

The Canon of Agreement has certain defects or disadvantages which may be indicated thus :—

(1) To generalize correctly, we must observe a large number of instances.

(1) The number of instances observed must be large enough to justify a generalization. If we observe a correspondence between an antecedent and a consequent in but a few cases, we cannot reasonably conclude therefrom that the two are causally connected. If, for example, we find a visitor at 4 P. M. on two or three successive occasions, we cannot hastily conclude therefrom that there is any connection between that hour and his visit. The coincidence may be purely accidental. In order to justify an inference that there is a causal connection between them, a more extended observation must be made; and if we really find that in numerous instances the individual pays visit only at 4 P. M., then we may be justified in supposing that there is a causal connection between that hour and

his visit, *i.e.*, in supposing that the individual finds that hour to be the most convenient for his visit.

(2) The Canon fails in the case of Plurality of Causes. If one and the same effect can be produced by different causes on different occasions, then the Canon of Agreement can never warrant us in inferring that a variable factor in the antecedent is not a cause. In the above symbolical example B may be the cause of *a* in the first case, D in the 2nd, and R in the 3rd. Thus, on the assumption of the doctrine of plurality of causes, we cannot conclude that B, D, or R is not the cause of *a*, because it is not uniformly preceded by any one of them. When, for example, a conjuror produces wonderful results by different tricks on different occasions, taking care to use his wand in each case, then the inference that his wand, and not the tricks, is the cause of the results is evidently not valid, though it is so suggested to the spectators. Likewise, a doctor may administer castor oil with rose syrup to a patient, mercury with rose syrup to another, and croton-oil with rose syrup to a third person, and the common result in all these cases is loose evacuation. From this evidently we are not justified in inferring that rose syrup is a laxative, though such an inference may be suggested to a layman. This defect is remedied, as we shall see, by the employment of the Joint Method or even by the Method of Difference.

(2) The Canon fails in the case of Plurality of Causes.

(3) The Method of Agreement being mainly a method of observation is subject to the limitations of observation. Its conclusion, therefore, cannot be

(3) It is not competent to distinguish Causation

done as an experiment

from Co-existence.

more than a mere probability or a tentative certainty or to feel the way unless it is verified by the negative instances. In fact this method fails to *prove* (but only suggests) causal connection between the constant factors in the antecedents and consequents. All that the Canon of Agreement shows is that the one (the constant antecedent) goes with the other (the constant consequent); but these two factors may be related simply by co-existence and not by causation, in which case an inductive generalization would be highly precarious. Or, the common antecedent and the common consequent may be the co-effects of some other cause, as in the case of day and night. "It was a general belief at St. Kilda," says Dr. Paris, "that the arrival of a ship gave all the inhabitants *colds.* Dr. John Campbell took a great deal of pains to ascertain the fact, and to explain it as the effect of effluvia arising from human bodies; the simple truth, however, was, that the situation of St. Kilda renders a north-east wind indispensably necessary before a stranger can land—the wind, not the stranger, occasioned the epidemic." (*Pharmacologia*, p. 89.) We see, then, that the Canon of Agreement, though often employed by common people, is not competent to prove a causal connection, which lies at the root of all valid inductive generalizations.

§ 5. (II) The Canon of the Joint Method of Agreement in Presence and in Absence.

Enunciation of the Canon.

If (i) two or more instances in which a phenomenon occurs have only one other circumstance (consequent or antecedent) in common, while (ii) two or more

Method of double agreement.

instances in which it does not occur have nothing else in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ throughout (being present in the first set and absent in the second) is the effect or the cause of the phenomenon, or causally connected with it.

Illustration.

Agreement in Presence.	*Agreement in Absence.*
*A*BC	BCD
*a*bc,	bcd,
*A*DE	DEF
*a*de,	rst,
*A*KL	KLM
*a*kl.	klp.

This Canon implies the double application of the Method of Agreement, *viz.*, Agreement in Presence and Agreement in Absence. It implies that whatever is present in several observed instances with the presence of a phenomenon, and is absent in several observed instances with its absence, is causally connected with the phenomenon. It is, accordingly, applied where there are two sets of instances differing only in one antecedent and one consequent, which are uniformly present in the instances of one set and uniformly absent from the instances of the other. When this is the case, we naturally suspect that the antecedent and the consequent which are present in the positive instances and absent from the negative ones are related either as cause and effect or by some bond of causation which chains them together. Thus, in the symbolical example given above, we find in the first set of instances *A* to be common to all

the antecedents and *a* to be common to all the consequents. Hence we naturally suspect that *A*, which is uniformly followed or accompanied by *a*, is its cause or indispensable condition. This suspicion is further strengthened by the second set of instances constituting agreement in absence. In the second set we find that *A* is uniformly absent from all the antecedents, and *a* is likewise absent from all the consequents. This agreement in their absence naturally inspires in us the belief that they are causally connected, so that the one being absent the other also is found to be so.

It is more conclusive than the Canon of Agreement.

The Joint Method is more conclusive than the Method of Agreement alone. The negative instances support the prior suspicion excited by the positive ones. But, like the Method of Agreement, the Joint Method fails to distinguish between causation and co-existence or the co-effects of a common cause. But the doctrine of Plurality of Causes does not invalidate this Canon or Method, for any one of the supposed causes may be present in the negative instances, without producing the effect. B, C, D, E, K, and L, for example, cannot be causes, in the above illustration, since they are present here or there in the negative instances without giving rise to the effect *a*. We often employ this method in common life to discover a causal relation. It is thus that we determine the force of personality in any case or the wholesome or unwholesome character of some diet or climate. If, for instance, we find that whenever we take any particular article of food or live in a particular

Though it cannot distinguish between Causation and Co-existence, yet it is not vitiated by Plurality of Causes.

Illustrations.

place, we suffer from indigestion or cold, and as often as we refrain from taking the article of food or we migrate to some other place, we do not so suffer, we naturally connect our illness with the article of food or the place. When in a boat-race or in a foot-ball match, a boat or a party wins when a particular individual is the strokesman or goal-keeper, while it loses, when that individual retires or is absent, inspite of other changes in the crew or party, this method indicates that victory is due to the influence of this individual. Similarly, it was believed that the successes of Napoleon were to a great extent due to the force of his personality, as his presence in battles resulted in victories, while his absence, in reverses. The importance of cavalry in war is likewise proved by the successes of Hannibal, Cæsar, and Napoleon when good cavalry was employed, and their reverses when cavalry degenerated or was wanting.

It may be mentioned here that the Joint Method is implied in the Method of Agreement, since the study of the instances of positive agreement naturally suggests also the instances of negative agreement or agreement in absence. What, however, the Joint Method expresses is that these two sets of instances—positive and negative—are explicitly examined and compared, with a view to a surer conclusion regarding a causal connection, as distinguished from an implicit reference to the negative instances involved in what is called the Method of Agreement.

Enunciation of the Canon.

§ 6. (III) The Canon of Difference. *If an instance in which a phenomenon occurs and an instance in which it does not occur, have every other circumstance in common save one, that one (whether consequent or antecedent) occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable condition of the phenomenon.*

What this Canon implies is that when we have a pair of clearly defined instances of succession, agreeing in all respects except an antecedent and a consequent, which are present in one case but absent in the other, then we are led to think that the antecedent and the consequent are causally connected. When the presence of an agent, therefore, is followed by the appearance, and its absence by the disappearance, of a certain event (other conditions remaining the same), then it is generally thought that there is a causal connection between the agent and the event. Thus,

Illustrations.

*A*BC	BC
*a*bc.	bc.

Here we notice in the first case that *A*BC are followed by *a*bc, while in the second case we find that BC are followed by bc. In the second case we find that *A* is absent from the antecedent, and *a* is absent from the consequent. Hence, we naturally conclude that *A* is the cause of *a*; for the two are uniformly present in the first, and they are uniformly absent from the second case. This Canon of Difference has thus its force even when it is employed as a method of observation. But

It is principally

its chief merit lies in its being employed as an instrument of experiment. When, having observed that A is followed by a, we suspect that they are causally connected, and then we have recourse to experiment to verify our suspicion, the full force of this method is clearly illustrated. We remove, for example, A and we find that a also disappears; and this is a conclusive proof that A is the cause of a. We find, for example, that when we strike a bell placed in the receiver of an air-pump full of air, sound is produced; but as soon as the air is pumped out, there is no more any sound. We are, accordingly, led to think that sound is produced by the vibration of air. But, in order that this conclusive evidence may be forthcoming, the positive and negative instances should agree in all respects except one (*viz.*, the presence in the one case, and the absence in the other, of the supposed cause and the effect); otherwise the absence of the effect in the second case may be due to some other change. Hence, it is remarked that the Canon of Difference is peculiarly efficacious in establishing inductive generalizations, though it is often very difficult fully to satisfy the requirements of this Canon. When, for example, an individual suffers from ill-health in a particular place, and suspects that its climate is injurious to him, he may migrate to some other place and may be completely cured. From this he may infer that the climate of the first place was the real cause of his illness. But, such an inference is precarious, when other changes are simultaneously introduced,

employed in experiments, and is often very conclusive.

The requirements of this Canon are stringent.

such as those in diet and habits. It may thus be altogether erroneous to attribute his illness to climate, when it might have been due to previous diet or habits. (*Vide* Chap. XXX, § 6.)

This Method operates either by subtraction or by addition.

It may be mentioned here that the Method of Difference may operate either by subtraction or by addition. Thus, the withdrawal of an antecedent and the immediate disappearance of a consequent may as much rouse the suspicion of a causal connection between them, as the introduction of an antecedent and the immediate appearance of a consequent. Thus, when the sun is suddenly obscured by a cloud we infer not merely that the sun is the cause of light, but also that the cloud is the cause of darkness. Hence elimination or addition may reveal a causal link, provided the other circumstances remain the same. (*Vide* § 4.)

It is generally employed in practice.

We have said that all men are born logicians, more or less. There is a natural tendency even in children to exercise their intelligence aright for the attainment of truth. Thus, the canon of Difference, as the most cogent Experimental Method, is often employed even by children and rustics to determine a relation of causation. When a child, for example, finds that on shutting the eyes there is no visual experience, while on opening them there is, it naturally infers that the eyes are the organ of vision. Similarly, a cultivator attributes bad harvest to drought when he finds that, other circumstances being equal, there is good harvest when there is rain, while the absence of rain is attended by the failure of crop. If instances be

multiplied, and inference drawn from several cases of agreement in presence and absence, then the ground of inference is not the Method of Difference, but the Joint Method, as explained above. It may also be mentioned that the Method of Difference is not frustrated by Plurality of Causes. If, with the removal of a particular antecedent, a particular consequent also disappears, it proves that they are causally connected, whatever other causes might possibly exist.

Difference between the Joint Method and the Method of Difference.

From the above account we may easily gather the important points of difference between the Joint Method and the Method of Difference : (1) The Joint Method is generally employed in those sciences or spheres of inquiry where we have to depend on observation ; but the Method of Difference is pre-eminently experimental. The former is thus not so conclusive as the latter. (2) The Joint Method requires a large number of instances, both positive and negative ; but only two instances, one positive and the other negative, are sufficient for the Method of Difference. Thus, the former is comparatively laborious, while the latter rather easy when possible. (3) In the Method of Difference we prove our point directly by comparing the positive with the negative instance. But in the Joint Method the result is first obtained from the positive instances and then it is confirmed by the negative ones. Hence the Joint Method has sometimes been called the 'Indirect Method of Difference.'

Enunciation of the Canon.

§ 7. (IV) The Canon of Concomitant

Variations. *Whatever phenomenon varies in any manner whenever another phenomenon (consequent or antecedent) varies in some particular manner is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*

Exact correspondence in changes suggests causal connection.

As variations are changes in time, when any correspondence is repeatedly noticed between them, we naturally attribute it to causal connection and not to mere chance. Of course, extended observation and careful study are necessary before we are justified in arriving at such a conclusion; and the more definite and exact the correspondence, the surer the conclusion. "The illustrations of this law," as Jevons observes, "are infinitely numerous. Thus Mr. Joule of Manchester, conclusively proved that friction is a cause of heat by expending exact quantities of force by rubbing one substance against another, and showed that the heat produced was exactly greater or less in proportion as the force was greater or less. We can apply the method to many cases which had previously been treated by the simple method of difference; thus instead of striking a bell in a complete vacuum, we can strike it with a very little air in the receiver of the air-pump, and we then hear a very faint sound, which increases or decreases every time we increase or decrease the density of the air. This experiment conclusively satisfies any person that air is the cause of the transmission of sound." (*Elementary Lessons in Logic*, pp. 249-250.)

This Method of Concomitant Variations is applicable to those cases in which the Method of Difference *cannot* properly be employed: *viz.*, the cases in which the supposed potent factors are what are called 'Permanent Causes', *i.e*, causes of such a kind that their *total* elimination from the phenomena under investigation is not possible. Heat, cohesion, gravity, for example, are factors which cannot altogether be excluded from material bodies. Hence, in studying phenomena connected with these attributes, we are to apply this Canon. When we observe variations in intensity with regard to, say, 'heat' and also variations with regard to, say, 'volume,' we naturally infer a correspondence between 'heat' and 'volume' or 'density.' It is surmised that the brain is the organ of the mind, and this suspicion is based on this Canon. We notice the difference in weight or size of the brain in the case of different species or individuals, and we also notice variation in the degree of mental power. And, as we find differences in weight or size varying with degrees of mental power, we naturally surmise that the two are causally connected.

It is applied to cases where total elimination is not possible.

Illustrations.

These illustrations bring out the efficacy of this Canon in suggesting a hypothesis. Of course, all the Canons are methods of proof and not of discovery. (*Vide* Chap. I, § 13.) But, it should be remembered that concomitant variations materially help discovery when the phenomena under investigation are arranged in a graduated scale. "I have shown," observes Professor Ferri,

This Canon is an aid to discovery.

"that in France there is a manifest correspondence of increase and decrease between the number of homicides, assaults, and malicious wounding, and the more or less abundant vintage, especially in the year of extraordinary variations, whether of failure of the vintage (1853-5, 1859, 1867, 1873, 1878-80), attended by a remarkable diminution of crime (assaults and wounding), or of abundant vintages (1850, 1856-8, 1862-3, 1865, 1868, 1874-5), attended by an increase of crime." (*Criminal Sociology*, Eng. Trans., p. 117.) And he further illustrates how these crimes "in their oscillations from month to month display a characteristic increase during the vintage periods, from June to December, notwithstanding the constant diminution of other offences." (P. 77.)

This Canon may be viewed as a modification of Agreement or of Difference, according as other changes accompany or not the changes examined.

This Canon may be regarded as a modification of the Canon of Agreement or of the Canon of Difference, according as the changes under investigation are accompanied or not by other changes. Thus, if we find A B C followed by l m n, (2 A) B′C′ followed by (2 l) m′n′, (3 A) B″C″ followed by (3 l) m″n″, then we have to single out the connection between A and l in the midst of other variable factors (such as BC, B′C′, B″C″, m n, m′n′, m″n″), as in the case of the Method of Agreement. If, however, the attendant circumstances do not vary, but remain constant [such as ABC followed by $l$mn, (2A) BC followed by (2l) mn, (3A) BC followed by (3 l) mn], then the connection between A and l stands out distinctly, owing to the absence of other concomitant changes, as in the case of the

Method of Difference. In the former case, of course, the conclusion is not so certain as in the latter, since the correspondence in the variations of *A* and *l* may then be due, not to a direct causal connection between them, but to other causes, such as B or C associated with *A* and varying with it, or something else, owing to whose influence all these factors vary.

It should also be noticed in this connection that the Method of Concomitant Variations is applicable to cases where there are variations in quantity. Qualitative variations are measurable by the other Canons. Quantitative variations may be illustrated either in a direct or in an inverse form. For example, if a train travels 20 miles an hour when drawn by one locomotive, 40 miles an hour when drawn by two, and 60 miles an hour when drawn by three, we readily see the causal connection between motion and steam-power. Similarly, if the intensity of a current of water becomes $\frac{1}{2}$ when only one barrier is erected, $\frac{1}{4}$ when two are set up, and $\frac{1}{8}$ when three are employed, we easily discover the connection between a barrier and check on a water-current. Thus, correspondence in the variation of two phenomena, either in a direct or in an inverse form, naturally inspires in us a belief in their causal connection.

It is applicable to quantitative variations, manifested either in a direct or in an inverse form, qualitative variations being measured by the other Canons.

The Canon of Concomitant Variations is employed to bring out a causal connection prominently by what are known as the Graphic Method and the Method of Gradations. *The Graphic Method**

* The Graphic Method is often used in statistical inquiries, as it shows at a glance the fluctuations which we want to notice. The

The Graphic Method and

is the pictorial representation of the factors which vary together and which are suspected to be connected by way of causation. The Graphic Method may be illustrated by the following diagrams representing the correspondence between, say, the degree of heat and the quantity of volume, or say the prevalence of a disease (*e.g.*, plague) and change of season :—

FIG I.

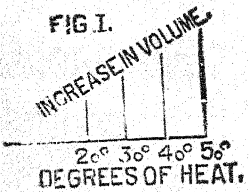
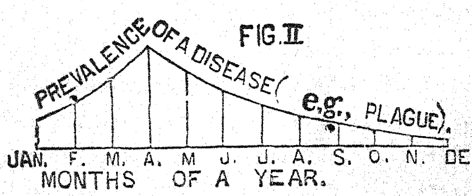
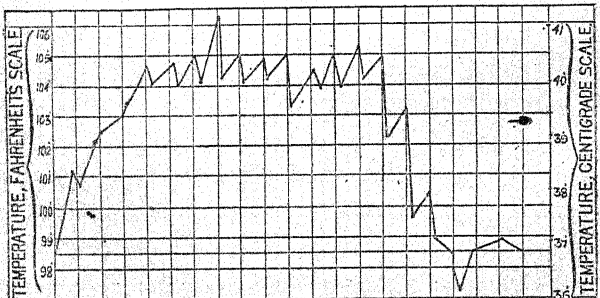


FIG II.



The following diagram represents also graphically correspondence between the variation in temperature and the lapse of time in the course of the typhus fever :—



abscissa or horizontal line shows variation in an agent or condition, called the *variable*, while the ordinates or perpendiculars indicate variations in the connected phenomenon, called the *variant*. See the diagrams on the next page.

The Method of Gradations or what is called *the Serial Method* consists in arranging in a series, or in a graduated scale, several objects illustrating variations in some fundamental attribute. For example, we may arrange human brains of different sizes or weights and notice corresponding variations in mental powers, or we may arrange the different modern languages in a series according to the degree of complexity and we may also notice corresponding variations in culture, and thus detect a causal connection between the two (*viz.*, the size or weight of the human brain and the degree of mental power in the one case, and the richness of language and the degree of mental culture in the other).* [*Vide* Chap. XXVI, § 2.] If we supplement such an inquiry by an examination of the brains and the mental powers of other animals or the study of languages in the different periods of history among different nations, then such a procedure is described as the *Comparative Method.*

the Method of Gradations or the Serial Method are based on this Canon.

The Comparative Method.

* Dr. Bosanquet writes—"I remember that a great many years ago I hardly believed in the stone-age tools being really tools made by men. I had only seen a few bad specimens, one or two of which I still think were just accidentally broken flints which an old country clergyman took for stone-age tools. This was to me then a mere guess, *viz.*, that the cutting shape proved the flints to have been made by men. And obviously, if I had seen hundreds of specimens no better than these, I should have treated it as a mere guess all the same. But I happened to go to Salisbury, and there I saw the famous Blackmore Museum, where there are not only hundreds of specimens, but specimens arranged in series from the most beautiful knives and arrow-heads to the rudest. There one's eye caught the common look of them at once, the better specimens helping one to interpret the worse, and the guess was almost turned into a demonstration, because one's eyes were opened to the sort of handwork which these things exhibit, and to the way in which they are chipped and flaked." (*Essentials of Logic*, pp. 143-144.)

The application of this Method becomes difficult when the correspondence is found to be irregular.

The employment of this Method is sometimes rendered difficult owing to irregularities in the correspondence of certain variations. In the case of Weber's law, for example, we notice that beyond certain limits the exact correspondence between the quantity of the stimulus and the intensity of the resulting sensation does not hold good; and we similarly find that water contracts as temperature falls till the freezing point is reached, after which it begins to expand. And the result of this method cannot be regarded as absolutely conclusive even in the case where two variations are found to have a definite numerical ratio between them, for they may both be but co-effects of a certain agent instead of being mutually related as cause and effect. In such cases, we must have recourse to wider observation and other methods to be sure of connection, suggested by this method.

Enunciation of the Canon.

§ 8. **(V) The Canon of Residues.** *Subduct from any phenomenon such part as previous inductions have shown to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.*

This canon implies that when any part of a complex phenomenon is known to be due to certain causes, the remaining or residual portion is due to some other cause besides the known.

It is applicable to complex cases.

It is applicable to complex cases in which an aggregate of several conditions or causes gives rise to an aggregate of several effects.

Illustrations.

If, for example, we observe ABC invariably and uncondi-

tionally followed by *abc*, and we know from previous inductions that *B* is the cause of *b* and *C* is the cause of *c*, then we can safely conclude that the residual or remaining antecedent *A* is the cause of the residual or remaining consequent *a*. Here the situation is a complex one, so that we cannot altogether separate *A* from its natural concomitants *B* and *C*, nor *a* from *b* and *c*. But, by subtraction, we infer that the remaining antecedent is the cause of the remaining consequent. Hence the Canon or Method is known as the Canon or Method of *Residues*.

This Canon rests on the assumption that the cause must be adequate to the effect.

This Canon rests evidently on the assumption that a cause must be adequate to the effect; so that when a part of a complex effect remains unaccounted for, we are led to attribute it to a cause other than the known causes of the other parts. If, then, we find some invariable antecedent associated with these causes, we naturally suspect it to be the cause of the residual phenomenon, as in the symbolical illustration given above. If, however, no such antecedent is presented to us, we are driven to hunt for it by analogies and past experiences, in order satisfactorily to explain the remaining effect. We thus see that the Method of Residues may be applied in two distinct, though allied, ways. Let us consider them separately.

Two prominent forms of the Method:

(1) To connect a residual consequent with a residual antecedent or antecedents.

(1) This method is applied to find out the agency of each of the several causes which combine to produce a complex effect, as indicated in the symbolical example given above. The enunciation of the Canon as given above from Mill, serves this

Illustrations.

purpose. When, for example, I smell a nosegay consisting of three kinds of flowers, the smells of two of which are known to me, then by this method I can refer the third smell to the third kind of flower. Similarly, if I weigh a bag with its contents, I can determine the weight of the contents by this method, when the weight of the bag is known to me. This method is also employed by Mill to infer the possible existence of *a priori* factors. He tries to explain our knowledge by reference to experience; and he mentions that if he failed thus to account for all the constituents of knowledge, then the residual factors might be explained by reference to *a priori* origin, the mind being the only other alternative source of knowledge. "The original elements," he writes, "can only come to light as residual phenomena, by a previous study of the modes of generation of the mental facts which are confessedly not original." (*Examination*, p. 179.)

(2) To discover the cause of a residual phenomenon.

(2) A more important application of the Method, however, is to discover an unknown cause of a residual phenomenon not explained by the given causes. Many examples cited by Mill and his followers illustrate this application, though it does not readily follow from the enunciation as given by them. To justify this use a distinct rule is needed, which may be enunciated thus :—*When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.* This form of the Canon is illustrated in the *discovery*

Enunciation of a Canon for this purpose.

of Neptune. It was noticed by astronomers that Uranus deviated a little from its calculated path. This residual phenomenon, *viz.*, the deviation, was accounted for by the hypothesis of an unknown body to whose influence the deviation was supposed to be due. The telescope was directed to the suspected part of the heavens with the result that Neptune was discovered as the cause of this deviation. Similarly, the *discovery of argon* was due to this method. It was found that nitrogen as found in the atmosphere was slightly heavier than nitrogen obtained from chemical sources. The cause of this difference in weight was supposed to be due to the presence of some other gas in the atmosphere, which was subsequently discovered to be argon.

Illustrations.

The above are the two chief applications of this method, though, no doubt, there is a third possibility of trying to discover the effect of a residual cause or antecedent. When, for example, we know ABC to be the antecedents, and also a to be the effect of A, and b, the effect of B, then we may be on the look out for the effect of C. Such a problem may, however, be solved easily by experiment, where possible, or by a study of other instances or combinations in which C is present.

A third form of the Method is to discover the effect of a residual cause.

Experiment easily settles such a question.

This method is, thus, valuable rather as a source of hypotheses than as an instrument of proving or testing them. Hence it may be said to be a method rather of *discovery* than proof. In fact it has been found particularly useful, as we have seen, in the discoveries of physical sciences.

This Method is valuable more as a source of hypotheses than as an instrument of proof.

Difference between this Method and the Method of Difference.

It is clear that this method very closely resembles the Method of Difference. But there is also a distinction between the two methods. In the Method of Difference the negative instance is obtained by direct experience, *i.e.*, by observation and experiment, whereas in the Method of Residues it is got by deduction from our prior knowledge of causal connections. Thus, unlike the Method of Difference, this method is based upon a previous knowledge of the laws of operation of different causes and a deductive computation of their total effect.

This method is especially suitable for quantitative investigations.

This method is sometimes called Deductive, as distinguished from the other Inductive Methods, but such a description is scarcely accurate, since all the canons are deductions from causation and all of them deduce conclusions from hypotheses, which they try to verify by an appeal to observation and experiment.

Like the method of concomitant variations, this method is specially concerned with quantitative investigations, its aim being to offer a complete and precise explanation of a fact. It is sometimes called a Deductive Method, as its procedure is prominently deductive. It first deduces the known consequences of known causes according to the known laws of their operation, and then, to complete the explanation, tries to deduce the known consequence of an unknown cause or, at times, the unknown consequence of a known cause, supposed to operate according to certain laws. Herein does it differ from the other Experimental Methods which proceed mainly by observation and experiment. But, inspite of this difference, we are not justified in calling this method Deductive, as distinguished from the other Inductive Methods, on the following grounds :—(1) The formation of hypothesis and deduction of consequences from it with a view to its verification are essential to all the

Methods. (2) Verification by an appeal to observation and experiment is necessary in all the Methods to set at rest any doubt about the correctness of our conclusion. (3) In a certain sense, all the Canons or Methods may be said to be deductive, for all of them follow immediately from the law of causation. In fact, the inductive canons, as we have seen, may be regarded as immediate inferences from the law of causation; and the application of these canons to particular cases to arrive at universal propositions may therefore be regarded as syllogistic inferences. (*Vide* § 1.)

As our knowledge advances and sciences become more deductive, there is greater room for the application of this Canon.

It may be mentioned in this connection that, as our knowledge progresses and the sciences become more and more deductive (*Vide* Chap. I, § 12), there is greater room for the application of this Canon. "It is by this process," says Sir John Herschel, "that science, in its present advanced state, is chiefly promoted. Most of the phenomena which nature presents are very complicated; and when the effects of all known causes are estimated with exactness, and subducted, the residual facts are constantly appearing in the form of phenomena altogether new, and leading to the most important conclusions." (*Discourse on the Study of Natural Philosophy*, § 158.)

§ 9. Characteristics and Uses of the Canons or Methods. It is clear from the above account that all the Inductive Methods are like fire purging away the dross of appendages and bringing out the golden link of causal connection, which binds the phenomena under investigation, in its

purity. Having explained in the above sections the different Methods one by one, let us now briefly indicate their characteristics and uses.

The method of Agreement. It is a method of observation.

The Method of Agreement. It is pre-eminently a method of observation in which we notice the phenomena under investigation as they are presented to us in the natural course of events. Besides the active regulation of attention to watch with care the phenomena observed, there is no active regulation of the phenomena themselves, such as we find in experiment. There is, thus, the difficulty of discovering whether the connected phenomena are related by way of causation or co-existence, for the most cogent test of causal connection (*viz.*, that, with the disappearance of the cause, the effect also disappears) cannot be applied to phenomena over which we have no control. We have also seen that the Method fails in the case of Plurality of Causes. (*Vide* § 4.) In spite of these disadvantages, this method is most commonly used both by the ignorant and the learned in arriving at empirical and scientific generalizations. There are phenomena, such as earthquakes, volcanic eruptions, hurricanes, movements of heavenly bodies, epidemics which can never be produced by experiment; and in such cases we must necessarily employ this Method. We have recourse to this method also when an agency is too dangerous to be experimented upon, as when we wish to determine the effects of misrule or of some violent poison.

It fails to distinguish causation from co-existence.

It is frustrated by plurality of causes.

It is generally employed to establish empirical and scientific generalizations.

It is applicable to phenomena beyond our control and to those which are too dangerous to be experimented upon.

The Joint Method. The

The Joint Method. It is more conclusive than the Method of Agreement, for the positive and

negative instances taken together produce a strong presumption in favour of a causal connection between the phenomena under investigation. As the positive instances agree only in the presence of the phenomena in question and the negative instances, in their absence, there is a strong probability of a causal link, specially when sequence is proved. We have seen that when the negative instances are exhaustive and eliminate all possible causes, the supposition of a plurality of causes is excluded. (*Vide* § 6.)

positive and negative instances together afford a more conclusive proof than either of them taken separately.

It is not vitiated by plurality of causes.

The Method of Difference. It is pre-eminently a method of experiment and is most cogent in proving the causal connection. The requirements of this method are, no doubt, very stringent; but, when they are satisfied, the proof is most conclusive. If the positive and the negative instance differ in nothing else than in the phenomena under investigation, then there is no doubt about their being connected by way of causation. "Agreement and Difference," says Bain, "can be easily compared as to their respective advantages and disadvantages. Agreement needs a large number of instances, but their character is not restricted. Any instance that omits a single antecedent contributes to the result; the repetition of the same instance is of use only as giving means of selection. Difference requires only one instance; but that one is peculiar, and rarely to be found." (*Logic*, II, p. 60.)

The Method of Difference. It is chiefly used in experiments and is very efficacious in proving causation.

Its requirements are stringent. Relative advantages and disadvantages of Agreement and Difference.

The Method of Concomitant Variations. It is applicable, as explained above, to those cases where

The Method of Concomitant Varia-

tions. It is applicable to phenomena which cannot be altogether eliminated.

the phenomena under investigation cannot be altogether eliminated. If, therefore, we observe that they rise or fall together in intensity, we are led to think that they are connected by causation. When living in a hot climate, we find that the rise of temperature is accompanied by depression of spirits, and that the greater the rise the greater the depression, then we naturally suspect that they are causally connected. To determine whether the phenomena thus varying together are related as cause and effect or as the co-effects of some other cause (as in the case of the rise of a river and the velocity of its current, both of which may be determined by rainfall), we should ascertain whether there is invariable succession or mere co-variation.

Concomitance may be illustrated either in a direct or in an inverse form.

Concomitance, we should remember, is illustrated either (*a*) in a direct or (*b*) in an inverse form. For example, (*a*) the greater the fuel, the greater the combustion; the greater the perseverance, the greater the likelihood of success; or (*b*) the higher the altitude, the less the rise of water in the common pump or of mercury in the barometer; the better the moral culture, the less the number of crimes. Both the forms are instructive in suggesting and proving causal connections.

This method often helps discovery.

We should also remember that this method often helps discovery, specially when concomitance is illustrated in an extreme form. "Very often," observes Bain, "we are not alive to a connexion of cause and effect till an unusual manifestation of the one is accompanied with an unusual manifestation of the other. We may be using some hurtful

article of food for a length of time unknowingly; the discovery is made by an accidental increase of quantity occurring with an aggravation of some painful sensation. This is one form of the efficacy of an Extreme Case; an efficacy felt both in science and in rhetoric." (*Ibid.*, p. 64.)

The Method of Residues. It is applicable to complex cases.

The Method of Residues. It is applicable, as shown above, to complex cases in which we are partially aware of causal connections. It presupposes, not merely a knowledge of the laws governing certain elements of the complex instance under investigation, but also the power of readily applying them to these elements, so that the law relating to the residual factor may correctly be suggested to the mind. This method thus involves abstraction and analysis in a prominent degree and requires also a rigid use of synthesis to preclude the possibility of alternative hypotheses relating to other factors which might be supposed to exist. This method, accordingly, is employed when some progress is made in inductive investigations and when a branch of study tends to become more and more deductive. (*Vide* Chap. I, § 12.)

It involves abstraction, analysis, and synthesis in a prominent form. It is employed when some progress is made in inductive investigations.

The different Methods ultimately rest on Agreement and Difference, which are regarded by Mill as fundamental.

§ **10. Unity of the Methods.** The several Inductive Methods, explained above, are not equally fundamental; they are but different applications of agreement and difference, which, as we have seen, underlie all inference. (*Vide* Chap. IX, § 1.) Hence, Mill regards Agreement and Difference as the two fundamental Methods or Canons which lie at the root of all inductive inquiries. The Joint Method, Concomitant Variations, and Residues are

not at all applicable, if certain factors or aspects do not vary, while others remain constant. In the case of Concomitant Variations, for example, though the factors which are supposed to be causally connected cannot altogether be eliminated, yet they vary in regard to their intensities: there is thus qualitative agreement with quantitative difference. And Agreement and Difference are really implicated in each other: in the case of Agreement, we find also room for Difference, so far as the variable factors are concerned; and, in the case of Difference also, room for Agreement, so far as the uniformity of sequence and the attendant circumstances are concerned. Mr. Read is inclined to hold that there is really one method at bottom, *viz.*, that of Difference. "In final analysis", he observes, "they are all reducible to one, namely, Difference; for the cogency of the method of Agreement (as distinguished from a simple enumeration of instances agreeing in the coincidence of a supposed cause and its effect) depends upon the omission, in one instance after another, of all other circumstances; which omission is a point of difference." (*Logic*, p. 225.) It is, no doubt, true that there must be difference in identity in order to reveal the efficient, as distinguished from the inert, factors in any case: we cannot infer a general relation by Agreement, if the several instances do not vary in respect of antecedents and consequents which are found to be accidental. But it is scarcely correct to hold that Difference, and not Agreement, is the fundamental Method.

Agreement and Difference are also implicated in each other.

Mr. Read holds that Difference is the fundamental method:

but though Difference is essential to elimination,

yet from mere Difference we

From mere Difference we can never infer anything. (*Vide* Chap. IX, § 1.) Difference merely brings out prominently the fundamental agreement in the several instances, as the background in painting gives prominence to a figure. Agreement and Difference ultimately rest on the fundamental intellectual functions of assimilation and discrimination, which underlie all mental processes whether logical or not; and thus the two methods are very closely connected. But if we be disposed to trace them to a single principle, then Agreement, Identity, or Consistency would seem to lie at the root of all inference or truth. (*Vide* Chap. II, § 11.)

cannot infer anything.

Difference only reveals the fundamental Agreement.

Agreement, Identity, or Consistency lies at the root of all inference.

It may be mentioned in this connection that the different Methods or Canons may all be applied to a case to determine a causal connection: some of the Methods may fail, some may be partially successful, while the rest may confirm the suspicion excited by the other Methods. Causal connection can scarcely be conclusively established by one Method alone. Even the Method of Difference, supposed to be so very convincing, leaves room for doubt, whether there may not be other influences owing to which both the antecedent and the consequent appear and disappear together. This doubt can only be removed by the employment of the other Methods. The imperfections of the different Methods are, to a great extent, neutralized when all of them are directed to one and the same inquiry: what is left doubtful by one Method may then be settled by another. When

All the Inductive Methods may be applied to a case to establish a law or causal connection.

all of them are focussed on the same investigation, they generally yield sufficient light to dispel all darkness and thus to reveal the causal connection which may subsist between certain factors forming the subject-matter of inquiry. The truth of these remarks will be clear from the following section.

Concrete examples:

§ 11. **Examples of the Methods.** Having now studied the different Inductive Methods or Canons, by which we must test the validity of every inductive generalization, let us next try to apply our theoretical knowledge to some concrete cases to prove the uses of these Methods or Canons.

(I) An inquiry into the cause of 'Dew'. It involves the following steps:

(1) The phenomenon of '**Dew**' is ordinarily taken as illustrating more or less perfectly the several conditions of inductive inquiry, as indicated in Chapter XVI, § 5.

(*i*) Careful observation of the phenomenon to be explained (*viz.*, what is known as dew).

(i) *Observation.* The first step necessary for the discovery of the cause of 'dew' is to observe carefully the phenomenon which we try to explain and to mark it out from other analogous phenomena. We observe the formation of 'dew' in the form of moisture; and we distinguish this from other similar phenomena like 'fog', 'mist,' or 'rain' by the fact that 'dew' is the spontaneous deposition of moisture on a surface when there is no visible wetness in the atmosphere.

(*ii*) Its definition

(ii) *Definition.* This leads to the definition of 'dew', which conveys a precise knowledge of the phenomenon under investigation.

and (*iii*) analysis.

(iii) *Analysis.* The phenomenon to be explained being an effect, we cannot possibly have

recourse to experiment. So we must observe with care the phenomenon and its concomitant circumstances, with a view to frame some hypothesis about its cause. The concomitant circumstance, which at once attracts our notice, is the nightfall, involving coldness and darkness. This may be regarded as an analysis of the situation essential to the formation of a hypothesis.

(iv) *Framing Hypotheses.* When by analysis we find that darkness and coldness are generally the uniform conditions of the production of 'dew,' we naturally suspect one or both of these factors to be the cause of 'dew.' Thus, we form hypotheses to account for the phenomenon.

(*iv*) Starting hypotheses.

(v) *Exclusion of Rival Hypotheses.* But the hypothesis in favour of darkness is excluded by the fact that 'dew' is sometimes deposited before nightfall and also by the fact that 'dew' is not formed every night. Thus, we are driven to the only alternative that coldness is perhaps the cause of 'dew.'

(*v*) Exclusion of rival hypotheses by elimination.

(vi) *Application of the Inductive Methods* or *Canons.* Let us test this hypothesis by applying the several Inductive Methods or Canons.

(*vi*) Application of the Inductive Methods :

(1) When we apply the *Method of Agreement,* we find that the object on which 'dew' is deposited is colder than the surrounding atmosphere. If we test it by using the thermometer, we also find that the temperature of a dewed object is less than that of its surrounding atmosphere. This illustrates Agreement in Presence : whenever a surface is dewed, it is colder than the air around it. But,

(1) Agreement ;

Agreement in Absence is not always illustrated : there may be cases of surface coldness without any deposition of 'dew.'

(2) Joint Method ;

(2) The *Joint Method* (which is but a combination of Agreement in Presence and Agreement in Absence) thus fails to discover the cause in this case.

(3) Difference;

(3) Let us now apply the *Method of Difference* We observe in the same night that some objects are dewed while others are not ; and we find also that the one class of objects is colder than the other. From this we naturally suspect that coldness is perhaps the cause of the deposition of 'dew.' But such a suspicion is unwarrantable, for the objects which differ in temperature (*e.g.*, a blade of grass and a piece of metal) also differ in many other respects. And thus an uncertainty is left as to which of the varying circumstances is really the cause of 'dew.'

(4) Concomitant Variations in respect of

(4) Let us now see whether the *Method of Concomitant Variations* is applicable. And we find that the method is illustrated in three ways here :—

(a) material,

(*a*) When we take into account the *character of the material*, we find that all objects are not equally dewed. If, for example, we expose wood, metal, glass, cloth, *etc.*, in the same night, we find that they are dewed in different degrees ; some are dewed more and some, less. And we find that the degree of the deposition of dew depends inversely on the conducting power of the object dewed ; good conductors are less dewed and bad conductors are more dewed ; and the quantity of 'dew'

depends on the degree of the badness of conducting power.

(*b*) If we compare objects by reference to their *surfaces*, we find that rough and black surfaces are better dewed than smooth and white ones. And it is known that rough and black surfaces are good radiators of heat. So, we conclude that the quantity of 'dew' varies directly with the radiating power of a substance. (*b*) surface,

(*c*) If we compare objects by reference to their *texture*, we find that compact bodies are less dewed than loose ones. And we know that compact bodies are good conductors of heat and loose bodies are bad conductors. This fact may be connected with (*a*). And, if we take all these facts into consideration, we find that the degree of deposition of 'dew' depends always on the coldness of the surface. In the case of bad conductors, and so in the case of loose bodies, the surface becomes cool sooner, because the inner heat is not quickly transferred to the surface owing to the defect of conducting power. Thus, the Method of Concomitant Variations shows that the deposition of 'dew' is always connected with the coldness of the surface. But here another difficulty presents itself affording an opportunity for the application of the remaining canon. and (*c*) texture;

(5) We find that the deposition of 'dew' does not always depend on either the absolute or the relative coldness of a surface. Though the difference between the atmospheric temperature and the temperature of a body be the same in two different (5) Residues.

nights, yet we find that 'dew' may be deposited in one night while not in the other. This evidently indicates that mere surface cooling is not the only condition of the deposition of 'dew'; there must be some other residual phenomenon which also determines such deposition. This residual phenomenon was easily suggested by the speculation of Dalton, who had propounded his theory of Aqueous Vapour or the Atmosphere of Steam, indicating the quantity of vapour which might be sustained in air. It had been proved by him that the quantity of aqueous vapour sustained in air varies with its temperature. The maximum quantity of vapour which may be supported in air at a temperature of 80° is said to be equivalent to one inch of mercury; and an amount equal to half an inch is supported at a temperature of 59°. Though the maximum quantity of vapour sustained in the atmosphere is thus determined by its temperature, yet we find that the air is not always saturated up to the extreme limit. In such cases, though there may be a fall in the temperature of the air owing to its contact with a cool surface, yet there may not be the conversion of vapour into water (*i.e.*, the deposition of 'dew'), as the lowered temperature may still be able to sustain the comparatively less quantity of vapour. When, however, the temperature falls below the saturation point, then—and then alone—there is the formation of 'dew.' Thus, the deposition of 'dew' on a body would depend not merely on the coldness of its surface but also on the quantity of vapour contained in the surrounding atmosphere.

Dalton's Theory of Aqueous Vapour.

We see, then, that the several Inductive Methods have proved a connection between 'dew' on the one hand and surface cooling and atmospheric humidity on the other. That this connection is of a causal character is proved by two circumstances :— Causal connection is proved by

(1) Atmospheric humidity and coldness of surface are known to precede and the deposition of dew is known to follow. (1) sequence

(2) Moreover, the quantitative aspect of the causal connection shows that there is equivalence in the transfer of energy. We know that water is converted into vapour by the application of heat; and it follows that the withdrawal of the heat leads to the reconversion of vapour into water. Hence we conclude that the humidity of the atmosphere and the coldness of surface are the conditions or cause of the deposition of 'dew,' which is the effect. and (2) the transfer of energy.

(II) The following example taken by Mr. Ryland from the *Pall Mall Gazette* of the 15th October, 1883, may also be mentioned in this connection as illustrating the Inductive Methods :— (II) An inquiry into the cause of an epidemic of typhoid fever.

"In August, 1883, **an epidemic of typhoid fever** occurred in Camden Town. The medical officer, Mr. Shirley Murphy, prepared a plan of the district, on which he marked all the houses which had been attacked. His scientific knowledge at once suggested to him a number of hypotheses as to the origin of the attack. Putting them briefly, they were (1) the Regent's Canal; (2) the water supply; (3) the sanitary arrangements The steps involved in it are— (1) Observation of cases. (2) Framing of hypotheses.

in the houses; (4) the milk supply. The use of his plan, and inquiry at the houses, showed that the first three hypotheses were invalid. The houses attacked were not usually near the canal. Two water companies supplied the district, and houses supplied by both companies were attacked with impartiality. Sanitary defects existed both in attacked houses, and in those which escaped the disease; while the sanitary arrangements in some of the attacked houses were perfect. So far the positive and negative Methods of Agreement (Joint Method) had been applied as a means of testing the hypotheses arrived at,—not, be it noticed, to suggest the hypotheses, which were largely due to deductive reasoning from general laws of hygiene.

(3) Exclusion of rival hypotheses by elimination.

(4) Employment of the Inductive Methods:

(*a*) the Joint Method;

"The fourth hypothesis remained. By the application of the Method of Residues, the case in favour of the fourth hypothesis was not *proved* (as Mill suggests) but perceptibly strengthened. Now, by a direct use of the Method of Agreement, "it was discovered that out of 431 persons attacked, 368 were definitely known to obtain their milk from one particular milkman, Mr. X., while the remaining 63 might well have indirectly obtained it from him also...Out of all the houses attacked, 78 per cent. received their milk from Mr. X." This use of the Method of Agreement was, however, probably twofold. After the first score or so of cases a strong suspicion was probably aroused in Mr. Murphy's mind that Mr. X.'s milk was the true *causa sine qua non*; while the

(*b*) the Method of Residues;

(*c*) the Method of Agreement.

subsequent cases were, it would appear, rather used by way of test, or verification, of the hypothesis so formed. Whether this was so, the report gives no information.

"Mr. Murphy now definitely verified his hypothesis by examining the shop and premises of Mr. X., and the farms from which Mr. X. obtained the milk. He succeeded in proving that one of these farms, near St. Albans, was infected with typhoid, and thus finally showed the adequacy and truth of his hypothesis. As the older logicians would have said, he proved that the typhoid fever infection in Mr. X.'s milk was a *vera causa*." (*Logic*, pp. 218-220)

(III) An inquiry into a preventive of small-pox.

(III) The determination of the **value of vaccination** as a prophylactic against small-pox well illustrates the application of the Inductive Methods. Dr. Jenner, who discovered the efficacy of vaccination as a protection against the distemper, was first led to inquire into the matter by a prevalent belief in Gloucestershire that persons contracting cow-pox in dairy farms enjoyed immunity from small-pox.* And his interest was specially roused by the casual remark of a young country woman, who came to him for surgical advice, that she could not possibly take the disease as she had had cow-pox before. (A) This prompted him to frame the hypothesis that some protection could be obtained against the malady by inoculation with cow-pox matter. (B) He then set about to verify

Prevalent opinion based on induction by simple enumeration.

(A) Framing of hypothesis.

(B) Its verification:

* This belief was, no doubt, due to general experience and was thus based on induction by simple enumeration.

(1) Experiment:

his hypothesis. (1) And he commenced with experiments. His first case of vaccination was that of a boy of 8 years, whom he inoculated with cow-pox matter taken from a sore on the hand of a dairy maid who had contracted the disease by milking cows suffering from cow-pox. But the conditions of the Method of Difference were not satisfied, as, besides vaccination, there might be many other unknown factors calculated to secure protection from small-pox. (2) Hence he next inoculated other persons, all of whom enjoyed immunity from the disease. This gave him an opportunity for applying the Method of Agreement. (3) And, when to these positive instances the negative instances of non-inoculation with greater liability to the disease were added, there was an opening for the Joint Method (involving Agreement in Presence and in Absence). The success of these experiments induced Dr. Jenner to believe that the protective influence of vaccination was complete and permanent. Subsequent experience proved, however, that this was untrue; and it thus afforded an opportunity for the application of the remaining Inductive Methods. (4) The residual phenomenon required to secure adequate and durable protection was found in the necessity of re-vaccination after an interval of about ten years. (5) And the Method of Concomitant Variations was also illustrated, as it was found that the degree of safety was proportioned to the degree of success in vaccination. Thus, the Royal Commission, appointed in 1889 to report on the subject, observes

Method of Difference not satisfied. Employment of (2) the Method of Agreement, (3) the Joint Method, (4) the Method of Residues, and (5) the Method of Concomitant Variations.

"The beneficial effects of vaccination are most experienced by those in whose case it has been most thorough. We think it may fairly be concluded that where the vaccine matter is inserted in three or four places, it is more effectual than when introduced into one or two places only, and that if the vaccination marks are of an area of half a square inch, they indicate a better state of protection than if their area be at all considerably below this." It may be mentioned in this connection that Princess (afterwards Empress) Victoria was the first member of the royal family who was vaccinated when she was barely 3 months old, which had the effect of greatly diminishing the prejudice against Jenner's discovery among ignorant people.

Inference aims at explanation by either the inductive or the deductive procedure.

§ 12. Inductive Methods as Methods of Explanation. The aim of all inference evidently is to remove our perplexities and to satisfy our inquisitiveness. We are ever trying to reduce our detached experiences to system either by tracing facts to principles (induction) or by illustrating principles by facts (deduction). In order to achieve our end we must in the first instance be sure of the character of the data—be they facts or principles—and then we must take due care to employ the appropriate means to connect them with what is calculated to satisfy our understanding. Thus, in the case of deduction we must first determine precisely the meaning and scope of a general truth before we seek to verify it by reference to concrete cases; and, likewise, in the case of induc-

We must first be sure of the data before drawing a conclusion.

tion, we must previously be sure of the exact nature of the facts to be explained before we try to discover the laws which govern them. We see, then, how the Inductive Methods necessarily involve the two prominent steps of careful observation and of no less cautious analysis and elimination with a view to arrive at a satisfactory explanation of the facts observed. We have already seen that observation and explanation are interconnected processes seeking to strengthen each other. (*Vide* Chap. XVII, § 10.) We are, therefore, never satisfied with the bare observation of facts, but we push on to find out their explanation: we proceed from the 'how' or the 'what' of things to their 'why'. The employment of the Inductive Methods, accordingly, implies a preliminary observation of facts (including experiment when possible) and a subsequent explanation of them by reference to the laws which the manipulation of the Methods necessarily leads us to infer. We should remember, however, that explanation is always relative to the character of the materials to be explained as well as to the capacity and attainments of the investigator. (*Vide* Chap. II, § 1.) We should thus never lose sight of the features of the objects observed to guide our inquiry and should also try to determine precisely their qualitative and quantitative aspects to render the inquiry definite and exact. And then we must examine carefully the steps and presuppositions involved in arriving at an explanation, so that there may not be any flaw in the process. We must also bear in mind that inference and

The Inductive Methods thus involve both Observation and Explanation.

Explanation is relative to materials and attainments.

Inference and explanation

explanation are necessary only to finite intelligence that tries to render the obscure clear by tracing the unknown to what is known. To the Infinite Intelligence all things are ever present and so nothing requires an explanation. (*Vide* Chap. I, § 4.) As Laplace has said—"An intelligence who for a single instant should be acquainted with all the forces by which nature is animated, and with the several positions of the beings composing it, if further his intellect were vast enough to submit these data to analysis, would be able to include in one and the same formula the movements of the largest bodies in the universe and those of the lightest atom. Nothing would be uncertain for him: the future as well as the past would be present to his eyes."

are necessary only to finite intelligence.

§ **13. Difficulties in Induction.** It has been urged that the Inductive Principles and Methods are rather of an ideal character scarcely applicable to the actual study of facts, which are often complex and at times even subtle in their composition. The difficulties in the inductive procedure are believed to be (1) partly subjective and (2) partly objective.

It is urged that the Inductive Principles and Methods are not competent to solve practical difficulties.

The difficulties in the inductive procedure are supposed to be (1) partly subjective and (2) partly objective.

(1) The subjective difficulty is connected with the nature of our faculties and senses which are regarded as too coarse and obtuse to penetrate the subtleties of Nature. To determine, for example, the true cause of a malady or the proper effect of a remedy is not always an easy task.

(1) The subjective difficulty is said to be due to the imperfections of our faculties and senses.

(2) The objective difficulty is connected with the facts themselves which we are called upon to

(2) The objective

difficulty is said to be due to the complexity of facts and the intricacy of their relations.

examine for inductive generalizations. The simplicity and isolation of facts and circumstances, required for inductive research, can scarcely be secured in practice. It is difficult, for example, to separate the effects of legislation from those of social opinion in order to determine their respective values. Thus, it is said that in actual experience facts are not so simplified or arranged as antecedent and consequent, cause and effect, as to render the application of the Methods possible. For example, the conditions of the Method of Agreement that 'the instances must have no circumstance in common but one' and of the Method of Difference that 'the two instances should differ in the presence of only one circumstance' can be fulfilled only in an ideal rather than in an actual world. Hence, Dr. Whewell says "The Methods take for granted the very thing which is most difficult to discover, the reduction of the phenomena to formulæ such as are here presented to us." (*Phil. of Discovery*, p. 263.)

It may be mentioned, however, that the imperfections of our faculties and the complexity of phenomena are not so great as to baffle inductive inquiry or generalization.

It may be replied, however, that we must be satisfied with the degree of isolation or simplicity observable by us. Such isolation or simplicity justifies generalizations which are aids to knowledge as well as to practice. And a like remark applies to the so-called defects or imperfections of our senses and faculties. We must be content with such knowledge as we may acquire through them. To emphasize the defects of our constitution as so very great as to exclude all definite knowledge, is merely to advocate a degree of

scepticism which does away with all knowledge and theory alike.

§ 14. **Exercises.**

1. What do you understand by the Experimental Methods? Why are they so called? Indicate their aim and the uses to which each is appropriate.

2. Show that the Experimental Methods are deductions from the Law of Causation. Point out the particular aspect of Causation on which each of them is based.

3. Give a critical exposition of the special function and conclusiveness of each of the Experimental Methods and reduce them to two fundamental methods of Elimination.

4. Illustrate by examples any two of Mill's Experimental Methods.

5. Explain and illustrate the Method of Difference, showing how it is oftener than any other the basis of ordinarny inferences. How is it related to the Method of Concomitant Variations?

6. In what does the superiority of the Method of Difference over the Method of Agreement consist? What is the necessity for recourse to a third or Joint Method, and what is the nature of that Method?

7. Explain the Canon of the Double Method of Agreement, and illustrate your answer by a concrete example. When is it necessary to employ this Method?

8. Expound, by a concrete application, the canon of the Method of Concomitant Variations, indicating its different forms. When is it necessary to employ the Method? Is it connected in any way with Elimination?

9. Are the Inductive Methods competent to prove cansation? Can they be properly called 'inductive'?

10. Indicate the uses and defects of the Method of Agreement. Explain and examine the remark of Bain, "Agreement for establishing an ultimate law is not the same as the Method of Agreement, in Mill's Canon, for establishing cases of causation." How can the defects of the Method be remedied?

11. Determine the characteristics and respective values of the different Canons of Induction. Are they traceable to a single principle?

12. Discuss the question whether the Inductive Methods may be viewed as mere weapons of Elimination. Examine the attempts of reducing them to one or two fundamental methods.

13. It has been said that it is probably the greatest merit in Mill's logical writings that he points out the entire insufficiency of what is called the Baconian Method to detect the more obscure and difficult laws of Nature. Explain what the Baconian Method is and in what respects Mill departs from it.

14. Illustrate the employment of the Experimental Methods by reference to a concrete case, say, an inquiry into the cause of dew.

15. 'It is in the comprehensive law of causation, itself once established by induction, that we have the instruments for eliminating causes and effects in the detail.' Explain this statement and illustrate it by examples.

16. Explain and illustrate the following terms :—Varying the Circumstances, Inductive Elimination, Plurality of Causes, Intermixture of Effects.

17. Explain and illustrate by a certain example the Method of Agreement. Point out the difficulties connected with the employment of this Method.

18. Explain how Plurality of Causes and Intermixture of Effects affect the application of the Method of Agreement. What advantage has the Method of Difference over the Method of Agreement, and what advantage has the latter over the former?

19. Analyse and examine the following arguments, indicating the methods employed to establish the conclusions :—

(1) As some comets and certain meteoric showers are found to have the same orbits, it is surmised that all meteoric showers are but the debris of disintegrated comets. When Biela's comet was missing, it was, accordingly, pre-

dicted that, when next due, it would be replaced by a meteoric shower; and this prediction was fulfilled.

(2) Schwabe, after repeated observations, found that the sun-spots always reached their maximum once in about ten years. Lamont similarly noticed that the range of the daily variation of the magnetic needle increased and decreased once in 10 years and 4 months. Sir Edward Sabine likewise observed that magnetic storms reached a maximum of violence and frequency in about 10 years, and marked the remarkable coincidence between such storms and sun-spots in respect of their maximum and minimum variations, whether in phase or duration. Hence it is supposed that the sun-spots are the causes of these magnetic disturbances.

(3) With the help of the microscope infusoria or animalcula were discovered by an observer; and these were supposed to have been spontaneously generated. A series of experiments were, accordingly, performed to verify its truth. Bottles were filled with the juices of meat extracted by boiling and were carefully corked and sealed with mastic. The closed bottles were then intensely heated and allowed to cool. As subsequently living germs were seen in the enclosed meat-juices, the experiment was believed to support the theory of spontaneous generation, since all such germs must have previously been killed by repeated heating.

Some suspected, however, that the experiments had not been conducted with sufficient care. Further experiments with different kinds of infusions gave, however, the same result. As, in these experiments, the infusions were enclosed in thin flasks and hermetically sealed and kept in boiling water for about an hour, all germs were evidently destroyed. And it was found that no infusoria subsequently appeared in them. But now it was said that such prolonged boiling had destroyed not merely the germs, but the germinative power of the infusions as well. But this objection was easily overthrown when it was found that the infusoria again appeared as soon as the infusions were exposed to air inspite of their previous intense and prolonged boiling.

These and analogous experiments tended to show that there is something in the atmosphere capable of producing life in nutrient fluids; but it was not known whether this something is solid, fluid, or gaseous. Helmholtz, however, proved afterwards that it must be solid, since it would not pass through a moist animal membrane as fluids and gases would do.

(4) Twenty-seven sterilized flasks containing an infusion of organic matter are opened in pure air on the summit of a mountain. It is found that putrefaction does not set in in any of them. Again, twenty-three similar flasks are opened in a hayloft. Almost all of them show signs of putrefaction after a lapse of three days. Hence it is inferred that floating particles in the air are the causes of putrefaction.

(5) Sir Humphry Davy found on decomposing water by galvanism, that there were present an acid and an alkali besides oxygen and hydrogen, the two components of water. Suspecting the additional portion of the effect to be due to the partial decomposition of the glass holding the water, he substituted gold vessels for glass, but without any change in the effect. He then used distilled water and found a marked decrease in the quantity of acid and alkali evolved. He next thought that the perspiration from the hands might account for these additional constituents; and he found that, by avoiding contact, their quantity was still further reduced. Thinking that the traces which were still left might be due to atmospheric impurities, decomposed by contact with the electrical apparatus, he put the machine under an exhausted receiver and found no more any trace of acid or alkali.

(6) It is found in a large majority of cases that high intellectual activity is attended with impaired health; compare, for example, the lives of doctors, lawyers, and journalists with those of the rustics, army men, and country gentlemen. Numerous instances may be cited in support of this view. Thus, Newton and Leibniz were mostly invalids; Clerk Maxwell died young after a life of ill-health; Darwin was scarcely well for three consecutive days in his adult life. The lives of Pope, Chatterton, Keats, Shelley, Byron,

Gibbon, Carlyle, De Quincey illustrate the same truth. It would seem, then, that the mind at its best is never found in a body that is at its best.

(7) It was previously believed that one and the same nerve was employed in sensation and motion, which was apparently supported by the fact that when any nerve was severed both sensation and movement became impossible in the part supplied by it. Sir Charles Bell, however, subsequently pointed out that the apparently single nerves were really bundles of several nerve-fibres and that it was inconsistent to suppose that one and the same nerve-fibre could carry impression *to* the brain and motor energy *from* it at the same time. He, accordingly, suggested that the distinct nerve-fibres performed distinct functions of sensation and motor innervation, which could easily be discovered if they were traced to their separate roots in the brain and the spinal cord. He then experimented on both the cerebral and spinal nerves. Of the cerebral nerves, he selected two—the portio dura (having one root) and the fifth pair (having two roots). On cutting the portio dura in a living animal, he found only motion of the connected limb lost. On cutting those branches of the fifth pair which arise from one root he found only sensibility lost, while, on cutting the branches which arise from both roots, he found both sensibility and mobility destroyed. In the case of the spinal nerves, which have two roots (an anterior and a posterior), he found that the irritation of the anterior root was followed by convulsive movements of the connected muscles, but such was not the case when the posterior root was irritated. It has been discovered also that in the case of partial paralysis either sensation alone or motion alone is lost.

(8) Linnets, when confined and trained with singing larks, abandon their natural song and adhere solely to the songs of the larks. It is thus surmised that birds learn to sing only by imitation, as men learn to use their speech.

(9) Able men have generally very bad handwriting, while good handwriting is frequently found in men doing

comparatively little mental work. Hence it is inferred that mental strain is the cause of poor penmanship.

(10) The great famine in Ireland began in 1845 and reached its climax in 1848. During this time agrarian crime increased very rapidly until in 1848 it was more than three times as great as in 1845. After this time it decreased with the return of better crops, until in 1851 it was only 50 per cent. more than it was in 1845. It is evident from this that a close relation of cause and effect exists between famine and agrarian crime.

(11) The mind must be a function of the brain, since any serious injury to the brain is always followed by the loss of consciousness.

(12) The flood was evidently due to the wrath of the goddess, since it began immediately after she had been slighted, and it subsided after propitiatory sacrifices.

(13) Moisture bedews a cold metal or stone when we breathe on it. The same appears on a glass of ice-water, and on the inside of windows when sudden rain or hail chills the external air. Therefore, when an object contracts dew it is colder than the surrounding air.

(14) With various kinds of polished metals, no dew is deposited; but with various kinds of highly polished glass dew is deposited. Therefore the deposit of dew is affected by the kinds of substances exposed.

(15) Scarlet poppies, scarlet verbenas, the scarlet hawthorn, and honeysuckle are all odourless, therefore we may conclude that all scarlet flowers are destitute of odour.

(16) Dr. Popper, a well-known physician on the continent, has been making some interesting observations regarding the stature of individuals and the relation that exists between height and talent and genius. The doctor finds that not only persons with considerable talent, but the geniuses of the world all have been, and are, of medium size or less. Among statesmen he points out Attila, Cromwell, Frederick II, Napoleon, Gambetta, Thiers—all of whom were of very small stature. He has discovered that while most small

men are small in stature because of the shortness of their legs they are really tall in the length of their bodies. This very fact, he thinks, is perhaps the secret of talent and genius—a good stomach, big heart and lungs in a big body—as they have a direct effect on the intellect. These organs help to feed the brain properly and make big men mentally.

(17) What better proof can we give of the power of the rain-god Mwari to cause rain than that recently at Rhodesia, after a prolonged drought, rain fell in torrents when on the advice of a rain-doctor a native of the place, named Mtegedi, was publicly burnt to death as an offering to the deity?

(18) It is known by direct experiment that for any given degree of temperature, only a limited amount of water can remain suspended as vapour, and this quantity grows less and less as the temperature diminishes. Therefore, if there is already as much vapour suspended as the air will contain at its existing temperature, any lowering of the temperature will cause necessarily a portion of the vapour to be condensed as dew.

(19) It is evident that the green colour of plants holds some necessary relation to light, for the leaves of plants growing in the dark, as potatoes sprouting in a cellar, do not develop this colour. Even when leaves have developed the green colour, they lose it if deprived of light, as is shown by the process of blanching celery and by the effect on the colour if a board has lain upon it for a long time. (Coulter.)

(20) Another indication that the green colour is connected with light may be obtained from the fact that it is found only in the surface region of plants. If one cuts across a living twig or into a cactus body, the green colour will be seen only in the outer part of the section. (Coulter.)

(21) If an active leaf or water plant be submerged in water in a glass vessel and exposed to the light, bubbles may be seen coming from the leaf surface and rising through the water. The water is merely a device by which the bubbles

of gas may be seen. If the leaf is very active, the bubbles are numerous. That this activity holds a definite relation to light may be proved by gradually removing the vessel containing the leaf from the light. As the light diminishes the bubbles diminish in number, and when a certain amount of darkness has been reached the bubbles will cease entirely. If now the vessel be brought back gradually into the light, the bubbles will reapper, more and more numerous as the light increases. (Coulter.)

(22) Vesalius, the founder of modern anatomy, found that the human thigh bone was straight, and not curved, as Galen, the great authority on the subject for over a thousand years, had asserted. Sylvius replied that Galen must be right; that the bone was curved in its natural condition, but that the narrow trousers worn at the time had made it artificially straight.

(23) When electricity was first discovered in the laboratory, the question naturally arose whether this was the same as manifested itself in the clouds. On further investigation, it was found that the effects of thunder and lightning, in the atmosphere, were mostly the same as produced by the electricity prepared in the laboratory. Lightning travels in a zigzag line, so does an electric spark. Electricity sets things on fire, so does lightning. Both melt metals, destroy life, and cause blindness. Pointed bodies attract the electric spark; and lightning also has been known to strike spires and trees and mountain tops. Hence it follows that lightning is but electricity travelling from one cloud to another, as does an electric spark from one substance to another.

(24) Sir Joseph Lister, the father of aseptic surgery, thus indicates the origin of his method:—"When it had been shown by the researches of Pasteur that the septic property of the atmosphere depended, not on oxygen or any gaseous constituent, but on minute organisms suspended in it, which owed their energy to their vitality, it occurred to me that decomposition in the injured part might be avoided without excluding the air, by applying as a dressing some material

capable of destroying the life of the floating particles." He, accordingly, first used carbolic acid for the purpose, and found that his wards in the Glasgow Infirmary, which used to be infected with gangrene, soon became the healthiest in the world, though other wards, separated only by a passage way, continued as unhealthy.

(25) "Koch found that, while guinea-pigs, mice, and other animals were killed by inoculation with anthrax, birds were not affected. This invulnerability had very much struck Pasteur and his two assistants. What was it in the body of a fowl that enabled it thus to resist inoculations of which the most infinitesimal quantity sufficed to kill an ox? They proved by a series of experiments that the microbe of splenic fever does not develop when subjected to a temperature of 44° Centigrade. Now, the temperature of birds being between 41° and 42°, may it not be, said Pasteur, that the fowls are protected from the disease because their blood is too warm? Might not the vital resistance encountered in the living fowl suffice to bridge over the small gap between 41°—42° and 44°—45°?...This idea conducted Pasteur and his assistants to new researches. 'If the blood of a fowl were cooled,' they asked, 'could not the splenic fever parasite live in this blood?' The experiment was made. A hen was taken and after inoculating it with splenic fever blood it was placed with its feet in water at 25°. The temperature of the blood of the hen went down to 37° or 38°. At the end of twenty-four hours the hen was dead, and all its blood was filled with splenic fever bacteria. But if it was possible to render a fowl assailable by splenic fever simply by lowering its temperature, is it not also possible to restore to health a fowl so inoculated by warming it up again? A hen was inoculated, subjected, like the first, to the cold water treatment, and when it became evident that the fever was at its height, it was taken out of the water, wrapped carefully in cotton wool, and placed in an oven at a temperature of 35°. Little by little its strength returned; it shook itself, settled itself again, and in a few hours was fully restored to health. The

microbe has disappeared. Hens, killed after being thus saved no longer showed the slightest trace of splenic organisms. There have been great discussions in Germany and France upon a mode of treatment in typhoid fever, which consists in cooling the body of the patient by frequently repeated baths. The possible good effects of this treatment may be understood when viewed in conjunction with the foregoing experiment on fowls. In typhoid fever the cold arrests the fermentation, which may be regarded as at once the expression and the cause of the disease, just as by an inverse process, the heat of the body arrests the development of the splenic fever microbe in the hen."

Division II.

AIDS TO INDUCTION.

CHAPTER XIX.

HYPOTHESES.

Hypothesis lies at the bottom of every inductive generalization.

§ 1. **Importance of Hypothesis in Induction.** Induction, as we have to tried to show, always rests on hypothesis. We observe a few instances resembling one another in certain important features, which suggest to our mind a law connecting them together. We then put our conjecture or hypothesis to test; and, if it is borne out by facts, it is accepted as a law or inductive generalization. "The discovery of a universal law," observes Lotze, "is always a guess on the part of the imagination made possible by a knowledge of facts. This knowledge is recalled to our memory by the resemblance of the given case to analogous earlier cases." (*Logic*, § 269.) Induction, as we have seen, is practically limited to the causal problem. When, however, we inquire into the cause of an effect, we cannot make the latter reproduce the former. We, no doubt, first try to find out what *is* the cause in such a case; but when we fail to do so, we try to conjecture what is the *probable* cause; and, on our failure even in this respect, we try to imagine what *may be* the *possible* cause.

Bacon's protest against hypotheses was directed against extravagant conjectures.

It should be remembered, however, that rash and unwarrantable conjectures, not based on facts, are not only useless but mischievous in tendency, since they often lead to fruitless inquiries and vain enterprises. It was against such random guesses that Bacon and Newton protested; and the remark of Newton '*Hypotheses non fingo*' [I do not make hypotheses] was really directed against them. As a matter of fact, we find that Newton himself framed several hypotheses to account for different classes of phenomena. The presence of a centripetal force (gravity) in the sun holding the planets in their places in the solar system, the presence of a similar force keeping the moon in its orbit, and the corpuscular theory of light were all hypotheses advocated by him which were subsequently proved or disproved by facts. Mere empirical compilation of facts is not enough: only relevant facts should be collected, and this is possible by bearing in mind the purpose or end of the collection. Hence the indispensable necessity of hypothesis in Induction. Thus, Dr. Venn rightly observes that, in all inductive generalizations, "There is first a stroke of insight or creative genius demanded in order to detect the property to be generalized, and possibly also to detect the class over which this property is to be generalized. In really original inductions this step may be one of the highest degree of difficulty. Indeed, except in the trite examples of the text books, which mostly deal with such inductions as have either been familiar for ages or at any rate have

had all these difficulties cleared out of their path, this requirement can scarcely ever be entirely evaded." (*Empirical Logic*, p. 351.)

Hypothesis is mere unverified conjecture, while Induction is proved hypothesis.

The difference between Hypothesis and Induction, accordingly, lies in the fact that the one is a mere supposition or assumption from an ascertained truth without adequate proof, while the other is a supposition tested and adequately proved by the Experimental Methods. When certain facts suggest to our mind a law or truth, it is a mere guess or hypothesis; but if it is verified by elimination and variation of circumstances as required by the Inductive Canons, it is an induction. "In proportion," says Bosanquet, "as you merely *presume* a causal connection, it is guesswork or pure discovery. In as far as you can *analyse* a causal connection, it is demonstration or proof; and for Logic discovery cannot be treated apart from proof, except as skilful guesswork. *In as far as* there is no ground, it gives nothing for Logic to get hold of—is mere caprice." (*Essentials of Logic*, p. 145.) Logic is, thus, directly concerned, not with the framing of hypotheses, but with their examination or verification. It supplies a systematic code of rules in the form of the inductive canons by means of which causal connections previously surmised are carefully examined. Whenever a causal connection is proved in some cases, it is believed to be true in all similar cases, and thus a step is taken towards scientific or inductive generalization. "It is quite true," say Dr. Venn, "that the so-called Methods of Inductive Enquiry

Logic as a science of proof furnishes canons or rules for testing the correctness of hypotheses.

do not in themselves, and necessarily, involve any reference to induction, but the generalization is nevertheless always held in view while we resort to them." (*Empirical Logic*, p. 352.) When, therefore, a hypothesis stands the test of the Inductive Methods, it is accepted as a law or induction; otherwise, it is rejected as but an idle speculation. Thus, a hypothesis simply gives a start to an inquiry, but cannot finally settle it; this is generally done by the employment of the Inductive Methods.

§ 2. Circumstances Favouring Discovery. We have seen that Logic is rather a science of proof than of discovery. (*Vide* Chap. I, § 13.) It supplies tests to determine the validity of our inferences, instead of furnishing hypotheses which help discovery.

Hypotheses are due to imaginative insight, which is the source of discovery.

Hypotheses, as mentioned before, are due to imaginative insight which leads to discovery—when connected with the power of accurate observation and sound abstraction separating the essential from the non-essential. We may, however, mention here some of the circumstances which favour discovery by helping the formation of reasonable hypotheses. The circumstances are:—

***Circumstances favouring discovery*:**

(1) Examination of a large number of similar instances.

(1) Examination of a very *large number* of similar cases. Such examination would naturally suggest to the mind the common features present in them all and would thus help it in the framing of hypotheses by reference to these.

(2) Examination of a few instances

(2) Examination of a few cases *with great care* and *attention*. Close examination may reveal

the points of community and thus suggest a hypothesis which otherwise might be missed. *with care and attention.*

(3) Examination of comparatively *simple or uncomplicated* cases. Such cases would readily suggest the points of similarity palpably present in them and would thus lead to an appropriate hypothesis.

(3) Examination of a few simple cases.

(4) Deductive reasoning—immediate or mediate—is at times a source of hypotheses, even when such reasoning is invalid. Thus, by simple conversion of 'All material bodies have weight,' we may be led to suppose that, perhaps, 'All bodies having weight are material.' Such a conjecture may subsequently be put to test and then accepted or rejected. Similarly, even wrong syllogistic arguments may suggest hypotheses. Thus, we may be led to think that John may be a murderer, because like murderers he flees from the scene of crime, or that particles of moisture are in the descending smoke, since it is heavier than air, as are the particles of moisture. Likewise, we may argue—John is clever, and John is a merchant, therefore perhaps all merchants are clever. It may be mentioned here that a generalization from a typical case or example (*Vide* Chap. XXII, § 6) usually assumes the form of an argument in the third figure. We may, for instance, conclude that 'Man has such and such a structure of the brain or of the heart', because 'Jones has a brain or a heart of such a structure and he is a man.'

(4) Deductive inference, even when invalid.

(5) The Method of Concomitant Variations, as shown above, is often efficacious in suggesting

(5) Employment of the Method of

Concomitant Variations.

causal connections, specially when the connected phenomena can be arranged in a graduated series. "To arrange phenomena in graduated series (if possible) in order to study them," observes Mr. Read, "is, perhaps, the most definite maxim in the Art of Discovery. If their causes are unknown it is likely to suggest hypotheses: and if the causes are partly known, variation of the character of the series is likely to suggest a corresponding variation of the conditions; as in investigating the development of the forelimbs of vertebrates or the natural history of clothes." (*Logic*, p. 220.)

Extreme cases of concomitant variations are often very efficacious.

It may be mentioned here that extreme cases of concomitant variations are generally more successful in suggesting hypotheses than the intermediate instances. If, for example, intense heat aggravates a disease very much, while feeble heat is attended with slight aggravation, then the cause of aggravation is very easily detected. Similarly, the effect of weather or climate, of filth or cleanliness, of good or bad rule is clearly brought to light when it operates in a prominent form. And this often leads to the formulation of definite hypotheses to be subsequently tested by a further appeal to facts.

(6) Use of Analogy.

(6) Analogy is often a source of hypotheses. When, for example, we try to explain a special group of phenomena, we observe similar phenomena in other provinces of Nature and try to frame a hypothesis by reference to these. To explain, for instance, the attraction of heavenly bodies we may study with care the attraction of

terrestrial ones; and, observing the gravitating force of the latter, we may be led to surmise that a similar force governs the former. If, likewise, we observe striking points of similarity between the bison and the buffalo, we may be led to suspect that they have the same nature and are therefore moved by the same tendencies. Thus, we may be inclined to think that what is true of one is also true of the other. (*Vide* Chap. XXII, § 5.)

§ 3. Character and Forms of Hypothesis. From the preceding remarks it is clear that hypotheses are conjectures made without proof or evidence. We never call a proposition which rests upon satisfactory proof, deductive or inductive, a hypothesis. Propositions are in a hypothetical stage so long as they depend only on guess and are not conclusively proved either by the inductive canons or by the rules of deduction. A hypothesis is, accordingly, defined by Mill as "Any supposition which we make (either without actual evidence, or on evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be, or at least is likely to be, true." (*Logic*, II, p. 8.) A hypothesis is said to be verified when facts or known truths are deduced from it, the deduction being an evidence in favour of the correctness of the hypothesis.

Definition of Hypothesis. (Mill.)

Hypotheses assume different forms by reference to their objects. Thus, there may be (1) hypotheses

Three Forms of Hypotheses:

about agents or causes, or (2) hypotheses about collocations or combinations of circumstances under which such agents act or operate, or (3) hypotheses about the laws or modes of their operation.

(1) Hypotheses about an agent or cause of a phenomenon.

(1) When, for example, we attribute malarious fever to marsh gas, a conflagration to incendiarism, or the loss of an article to theft, we suppose an agent to account for the phenomenon in question. Similarly, in science, ether is assumed to explain the phenomena of light; or Neptune, assumed to account for perturbations in the movements of Uranus.

(2) Hypotheses about a collocation or combination of circumstances under which a cause acts.

(2) When, likewise, we refer the success or failure of an undertaking to a happy or unhappy combination of agencies or circumstances, or we attribute the explosion of gunpowder to the contact of a spark with it, we suppose a collocation to explain the phenomenon under investigation. The Ptolemaic or Copernican system of Astronomy similarly supposes a collocation of heavenly bodies to account for the order of the solar system.

(3) Hypotheses about law or mode of operation of a cause.

(3) Sometimes our supposition refers to the way in which an effect is produced, as when we try to discover how a thief got access to a house or how he effected his escape. The mode of operation of the law of gravitation, of definite proportions, of musical harmony, or of relativity was at first a matter of hypothesis or conjecture, which has subsequently been verified by further investigations.

The three forms of hypotheses usually go together.

The three forms of hypotheses indicated above are seldom found in isolation; hypotheses with regard to one are more or less connected with those

of the rest. When we suppose an agent, we ordinarily suppose also the conditions under which it acts and the mode or law of its operation as well. Thus, in supposing a thief, we suppose also the circumstances under which he committed the theft and the way in which he succeeded in removing the articles stolen. The supposition of ether or of gravitation usually goes with that of the law of its operation as well as with that of the circumstances under which it acts. And if a cause in the abstract has no meaning, if it is always intelligible by reference to the conditions of its exercise and the mode of its operation, then the interconnection of the three forms of hypotheses is seen to be quite natural.

While Dr. Whewell over-estimates the importance of hypotheses, Mill under-estimates it.

It may be mentioned in this connection that Dr. Whewell, with whom induction is primarily concerned with discovery, lays great stress on hypotheses as essential to inductive inquiry. Whenever by imaginative insight we frame a hypothesis capable of accounting for a group of facts, we discover, according to him, a law, *i.e.*, arrive at an inductive generalization. Mill, however, rightly points out that the Experimental Methods must be carefully employed before an inductive generalization is reached; and Logic, according to him, is, as we have seen, essentially a science of proof. (*Vide* Chap. I, § 13 and Chap. XVI, § 3.) But Mill is disposed to underestimate the importance of hypotheses in inductive investigation, which he regards as resting simply on the Experimental Methods. We should not

forget, however, that there is room for the application of the Methods only when hypotheses have already been formed: an inductive inquiry is not an aimless act but a systematic procedure to prove or disprove a suspected connection, *i.e.*, a hypothesis previously started. Even Mill is forced at times to admit this, as when he speaks of the function of hypotheses in "suggesting observations and experiments" as "one which must be reckoned absolutely indispensable in science. Without such assumptions, science could never have attained its present state: they are necessary steps in the progress to something more certain; and nearly everything which is now theory was once hypothesis." (*Logic*, II, p. 16.)

§ 4. Conditions of a Valid Hypothesis. Hypotheses, as we have said, are due to imaginative insight; but imagination may be exercised either within reasonable limits or in an extravagant manner. To account for the disappearance of an article from a particular place we may suppose either that it has been mislaid, or that it has been removed by some person, or that it has been spirited away, or that it has melted in the air. All these hypotheses, however, are not equally reasonable. Certain tests are, accordingly, laid down in Logic by the application of which we may determine the legitimacy of a hypothesis. Some of these tests, however, are applied *when a hypothesis is being formed* to exclude the possibility of extravagant conjectures; while there are other tests which are employed *after it has been formed*, to see whether

it actually explains what it was previously conceived as capable of doing. The former are known as the *conditions* or *requirements*, while the latter as *proofs* or *evidences* of a true hypothesis. We shall confine our attention to the Conditions in this section and shall dwell on the Proofs in the next. A hypothesis *ought* to satisfy the following conditions before it can be entertained at all as deserving of proof :—

Conditions and Proofs of Hypotheses. *Conditions of valid hypotheses :*

(1) *A hypothesis must in the first instance be definite and verifiable.* To say, for example, in the above illustration that the object has somehow disappeared or that it has been carried away by spirits is to maintain an indefinite and vague proposition, which can never be put to any test. "To be verifiable," observes Mr. Read, "an hypothesis must be definite ; if somewhat vague in its first conception (which is reasonably to be expected), it must be made definite in order to be put to the proof." (*Logic*, p. 248.) This condition evidently implies that every admissible hypothesis must be capable of deductive proof and so of verification : we must be able to deduce conclusions from a hypothesis and compare them with facts in order that it may be deemed as acceptable. "Even if we could imagine," as Jevons observes, "an object acting according to laws hitherto wholly unknown it would be useless to do so, because we could never decide whether it existed or not. We can only infer what would happen under supposed conditions by applying the knowledge of nature we possess to those conditions. When we attempt

(1) A hypothesis should be definite and verifiable.

Hence every Hypothesis should admit of deductive proof and verification.

to explain the passage of light and heat radiations through space unoccupied by matter, we imagine the existence of the so-called *ether*. But if this ether were wholly different from anything else known to us, we should in vain try to reason about it. We must apply to it at least the laws of motion, that is we must so far liken it to matter. And as, when applying those laws to the elastic medium air, we are able to infer the phenomena of sound, so by arguing in a similar manner concerning ether we are able to infer the existence of light phenomena corresponding to what do occur. All that we do is to take an elastic substance, increase its elasticity immensely, and denude it of gravity and some other properties of matter, but we must retain sufficient likeness to matter to allow of deductive calculations." (*Principles of Science*, pp. 511-512.)

(2) Hypotheses should suppose real agents.

(2) *Hypotheses should generally have reference to real agents, i. e., such as are known to exist in Nature.* Hence some other evidence of their existence must be adduced than simply their relation to the facts to be explained. Newton's maxim that "only *veræ causæ* [true or real causes] are to be admitted in explanation of phenomena" is to be interpreted liberally and not rigidly in scientific investigations. As we are not omniscient beings, we can never expect to have a thorough knowledge of all the departments of Nature. To exclude, therefore, a hypothesis simply because it supposes an agent or cause not hitherto known, is to close the door against all discovery and

investigation. As De Morgan says, "The physical philosopher has frequently to conceive law which never was in his previous thought—to educe the unknown, not to choose among the known." (*Budget of Paradoxes*, p. 51.) Hence the term **vera causa** should be taken, not in the sense of a cause already known to exist, but in the sense of a cause which may reasonably be believed as existing and whose existence, therefore, does not involve a self-contradiction. *Veræ causæ*, says Herschel, "must be such as we have good inductive grounds to believe do exist in nature and do perform a part in phenomena analogous to those we would render an account of; or such, whose presence in the actual case can be demonstrated by unequivocal signs." (*Natural Philosophy*, p. 209.) Similarly, Dr. Bosanquet observes, "A *vera causa* is a thing, or occurrence in a thing, whose reality we are thoroughly convinced of from the necessity of reconciling observed data, and there is no reason in the nature of things why a single science or a single range of reality should not suffice to produce such conviction." (*Logic*, II, p. 159.)

Meaning of '*vera causa*.'

It follows from this that some latitude should be allowed to the exercise of imagination that it may be able to penetrate the mysteries of Nature. As the limits to knowledge are not the limits to existence, it is permissible at times to suppose occult and unknown agencies, such as ether or atoms, to account for known phenomena. It should be remembered, however, that the conditions of proof in such a case should be more stringent that

It is permissible at times to suppose new agents; but the conditions of proof should then be more stringent.

we may not land in a region of myths or fairies. "Some Hypotheses," says Bain, "consist of assumptions as to the minute structure and operations of bodies. From the nature of the case, these assumptions can never be proved by direct means. Their only merit is their suitability to express the phenomena. They are *Representative Fictions*." (*Induction*, p. 132.) It is thus that we explain heat to be a form of motion or light to be the vibration of ether, which can never be directly perceived but which enables us to explain satisfactorily the known phenomena of heat or light.

A hypothesis should, therefore, be unconditional.

It is evident from the preceding remarks that a hypothesis should be unconditional, *i. e.*, it should not rest on another hypothesis or conjecture. If this be the case, it would be hard to prove a hypothesis, as there would then be proof behind proof. In fact, a hypothesis in such a case would simply be 'an airy nothing' eluding every attempt to test it by facts.

(3) A hypothesis must be self-consistent and in harmony with the known laws of nature.

(3) *A hypothesis should not be self-contradictory; nor should it be in conflict with the known laws of Nature.* When, for example, an eclipse is attributed to the agency of a monster (*Rahu*) swallowing the sun or moon and then again ejecting it, or rain is explained by reference to the agency of a huge elephant (*Airavata*) pumping and scattering water by means of its trunk, the mode of operation is altogether inconsistent with the known laws of nature. Nature does not reveal anywhere that such a vast consequence as an eclipse or rain is produced in the way described here by a monster

or an animal that is huge and invisible at the same time. Such a hypothesis, therefore, is *prima facie* absurd.

(4) A hypothesis should be adequate to render a satisfactory account of the phenomena to be explained.

(4) *A hypothesis must be adequate to account for the phenomena under investigation.* "One of the most familiar instances of an *inadequate hypothesis*," remarks Fowler, "is the theory started by Voltaire, there is little doubt in irony, that the marine shells found on the tops of mountains are Eastern species, dropped from the hats of pilgrims, as they returned from the Holy Land. Such a theory would obviously be inadequate to account (1) for the number of the shells, (2) for the fact that they are found imbedded in the rocks, (3) for their existence far away from the tracks of pilgrims, to say nothing of the fact that many of these shells bear no resemblance to recent Eastern species, while none resemble them exactly." (*Induction*, p. 100.) Likewise, if we explain the disappearance of a box by reference to the agency of a mosquito, the explanation is deemed unsatisfactory, as the hypothesis is quite inadequate to account for the effect.

(5) In framing hypotheses we should not transgress the law of parsimony, *i. e.*, a hypothesis should be necessary and not superfluous.

(5) *We should not unnecessarily multiply the agencies required to explain the phenomena under inquiry.* This is known as the *Law of Parsimony*: it forbids us to assume more than what is necessary to account for a group of facts. If, for example, we can satisfactorily explain the mechanism of the universe by reference to a single Intelligent Principle, there is no necessity of supposing myriads of atoms so adjusted as to be capable of evolving the universe. The first two 'Rules of

Philosophising' given by Newton in his *Principia* imply this condition. He writes:—"*Rule I.* No more causes of natural things are to be admitted than such as are both true, and sufficient to explain the phenomena of those things. *Rule II.* Natural effects of the same kind are to be referred as far as possible to the same causes." (Book III.) From this condition of a valid hypothesis it follows that it should be necessary and not gratuitous. If a phenomenon can be satisfactorily explained by existing laws, there is no necessity of a hypothesis. If, for example, three faculties can explain mental phenomena, we need not assume more.

Briefly speaking, then, a hypothesis, to be valid, should be definite, verifiable, adequate, unconditional, necessary, and consistent with the entire system of knowledge.

(1) The best test is verification.

§ 5. Proofs of a Hypothesis. The proofs or evidences which go to establish a hypothesis as plausible or legitimate are the following :—

(1) *The best test or proof of a hypothesis is its verification* * : *if the consequences deduced from a hypothesis tally with facts, the presumption is in*

* It has been argued by some logicians that verification by an appeal to facts may be said to involve the fallacy of affirming the consequent in a hypothetical-categorical syllogism. Thus, it may be argued, it is said,—'If a man swallows prussic acid, he will die ; he is dead, and therefore must have swallowed the acid'. That this view is untenable is evident from the fact that the antecedent or hypothesis is not inferred to be true from the mere presence of a consequent or an effect, found apart from it ; on the other hand it is taken to be true only when the consequent or effect *is found to follow from it.* If we find a man dying or dead after he has swallowed prussic acid, then we may take the hypothesis to be valid, subject always, however, to further confirmation from other instances, involving variation of circumstances—so as to exclude other possible explanations.

favour of its truth. Of course, a hypothesis may temporarily explain facts, which on fuller examination may not support it. In such a case a hypothesis previously accepted is subsequently modified or rejected according to the character of further evidence.

In verification there should be an honest and exact estimate of facts.

We should remember in this connection that in the attempt to verify a hypothesis we should be actuated by a pure regard for truth, so that we should never try to interpret facts in the light of a hypothesis but should always try to test a hypothesis by reference to facts. The opposite tendency is not infrequently illustrated in the speculations of enthusiastic thinkers disposed to defend their views on any topic or subject. The following illustration of the belief in a supernatural agency causing dreams bears out the truth of this remark. "The ancients were convinced," writes Lecky, "that dreams were usually supernatural. If the dream was verified, this was plainly a prophecy. If the event was the exact opposite of what the dream foreshadowed, the latter was still supernatural, for it was a recognised principle that dreams should sometimes be interpreted by contraries. If the dream bore no relation to subsequent events, unless it were transformed into a fantastic allegory, it was still supernatural, for allegory was one of the most ordinary forms of revelation. If no ingenuity of interpretation could find a prophetic meaning in a dream, its supernatural character was even then not necessarily destroyed, for Homer said there was a special portal through which deceptive visions

passed into the mind, and the Fathers declared that it was one of the occupations of the dæmons to perplex and bewilder us with unmeaning dreams." (*History of European Morals*, I, p. 385.) We should not in this way try to reconcile facts with our hypothesis. We should, on the other hand, estimate facts at their proper worth and determine the validity of a hypothesis by reference to them. "A simple absolute conflict between fact and hypothesis," says Jevons, "is fatal to the hypothesis." (*Principles of Science*, p. 516.) He, accordingly, lays down the rule that "*Agreement with fact is the sole and sufficient test of a true hypothesis.*" (*Ibid.*, p. 510.) A very prominent example of verification in science is found in the discovery of Neptune, which was previously supposed to produce perturbations in the movements of Uranus. (*Vide* Chap. XVIII, § 8.)

Verification includes authentic testimony and valid deduction.

Verification, however, should be construed liberally and not merely in the narrow sense of what is supported by personal observation. If there be recorded trustworthy evidence, it may be taken into account in determining the value of a hypothesis. Thus, the hypothesis, formed from an examination of Mill's treatment of Logic, that his aim in writing the book was to supply a method for social investigations, is settled beyond doubt by a letter written by him to Miss Caroline Fox, in which he distinctly avowed that object. Similarly, if a hypothesis can strictly be deduced from a law already established, such deduction would amount to verification, since the law from which the hypothesis follows is supported by facts.

(2) Decisive instances should be observed to decide between conflicting hypotheses.

(2) *Decisive instances should be sought to show that the hypothesis framed in any case is the only acceptable one under known circumstances.* Such instances at once decide which of the several rival hypotheses started in any case is really to be accepted. As Ueberweg says, "One single circumstance, which admits of one explanation *only*, is more decisive than an hundred others which agree in all points with one's own hypothesis, but are equally well explained on an opposite hypothesis, which has originated from our opponent's side of the question." (*Logic*, Eng. trans., p. 513.) What is called a **Crucial Instance** is a case of this sort which at once terminates the conflict between contending hypotheses; and if such a case is obtained by experiment, it is called an **Experimentum Crucis.*** The fact of the aberration of light, for example, is taken as a decisive instance supporting the Copernican, instead of the Ptolemaic, system of Astronomy: if the earth be at rest, why should there be the phenomenon of aberration at all?† Similarly, the free passage of comets through space has disproved the presence

Meanings of Crucial Instance and Experimentum Crucis.

displacement of heavenly bo true position to observer.

Illustrations.

* The expressions have been borrowed from Bacon. Latin *crucis* or *crux* implies a cross which is generally used as a finger-post at the crossings of streets to indicate the right ways. A crucial instance or experiment is thus the fact observed or found by experiment which indicates the correct hypothesis to be chosen. The term, as Bacon says, "is transferred from the crosses (or finger-posts) which are put up in crossways to mark and point out different ways." (*Novum Organum*, Bk. II, Aph. 36.)

† Jevons writes, "Copernicus asserted, in opposition to the ancient Ptolemaic theory, that the earth moved round the sun, and he predicted that if ever the sense of sight could be rendered sufficiently acute and powerful, we should see phases in Mercury and Venus. Galileo with his telescope was able in 1610 to verify the prediction as regards Venus, and subsequent observations of

of the Crystalline Spheres of the ancient astronomers, thereby supporting the Copernican system. The *Principle of Interference* in the case of light likewise decides in favour of the undulatory, as distinguished from the corpuscular, theory of light. If light be due to vibrations of ether, then, when ethereal waves interfere with each other, there ought to be alternate bright and dark bands of colour, as the effects of the waves are heightened by addition or neutralized by counteraction. And Fresnel actually proved by experiment that the phenomena of interference are, as a matter of fact, produced when two streams of light come into collision with each other. "Fresnel's experiment," says Edser, "gives decisive evidence in favour of the wave theory of light. That light when added to light should produce darkness is incomprehensible on any theory of the material nature of light." (*Light for Students*, p. 323.) The following experiment relating to variation in the rate of movement of light with a variation in the density of its media may also be regarded as crucial, deciding in favour of the undulatory theory. "If the undulatory theory be true, light must move more slowly in a dense refracting medium than in a rarer one;

Mercury led to a like conclusion. The discovery of the aberration of light added a new proof, still further strengthened by the more recent determination of the parallax of fixed stars. Hooke proposed to prove the existence of the earth's diurnal motion by observing the deviation of a falling body, an experiment successfully accomplished by Benzenberg; and Foucault's pendulum has since furnished an additional indication of the same motion, which is indeed also apparent in the trade winds. All these are crucial facts in favour of the Copernican theory." (*Principles of Science*, p. 522.)

but the Newtonian theory assumed that the attraction of the dense medium caused the particles of light to move more rapidly than in the rare medium. On this point, then, there was complete discrepancy between the theories, and observation was required to show which theory was to be preferred. Now by simply cutting a uniform plate of glass into two pieces, and slightly inclining one piece so as to increase the length of the path of a ray passing through it, experimenters were able to show that light does move more slowly in glass than in air. More recently Fizeau and Foucault independently measured the velocity of light in air and in water, and found that the velocity is greater in air." (Jevons, *Principles of Science*, p. 521.) "A crucial experiment," as Jevons points out, "must not simply confirm one theory, but must negative another; it must decide a mind which is in equilibrium, as Bacon says, between two equally plausible views." (*Ibid.*, p. 519.) Thus, a *crucial instance* or *experiment* is a fact found by observation or experiment, indicating which of two rival hypotheses should be accepted, just as a finger post at a crossing indicates which of the two roads that cross each other is to be taken.

(3) Exclusive sufficiency of a hypothesis to explain facts and concomitant circumstances.

(3) *In the absence of direct verification, the exclusive sufficiency of a hypothesis to account for the appropriate phenomenon and its attendant circumstances may be accepted as a reasonable proof.* We should remember that in such a case there should be no other way of explaining the phenomenon in question. Thus, in circumstantial evidence we

accept such a proof, when direct evidence is wanting. (*Vide* Chap. XXI, § 11.) Similarly, the supposed cause of erratic boulders or huge blocks of stone scattered over many parts of Northern Europe is ascertained in this way. The composition of these blocks clearly shows that once they were parts of hills to the northward of their present site. "They must, therefore, have somehow been detached and transported to where we now find them. How? One old explanation is that they were carried by witches, or that they were themselves witches accidentally dropped and turned into stone. Any such explanation by supernatural means can neither be proved nor disproved. Some logicians would exclude such hypotheses altogether on the ground that they cannot be rendered either more or less probable by subsequent examination. The proper scientific limit, however, is not to the making of hypotheses, but to the proof of them. The more hypotheses the merrier : only if such an agency as witchcraft is suggested, we should expect to find other evidence of its existence in other phenomena that could not otherwise be explained. Again, it has been suggested that the erratic boulders may have been transported by water. Water is so far a *vera causa* that currents are known to be capable of washing huge blocks to a great distance. But blocks transported in this way have the edges worn off by the friction of their passage : and, besides, currents strong enough to dislodge and force along for miles blocks as big as cottages must

have left other marks of their presence. The explanation now received is that glaciers and icebergs were the means of transport. But this explanation was not accepted till multitudes of circumstances were examined all tending to show that glaciers had once been present in the regions where the erratic blocks are found. The minute habits of glaciers have been studied where they still exist : how they slowly move down carrying fragments of rock ; how icebergs break off when they reach water, float off with their load, and drop it when they melt ; how they grind and smooth the surfaces of rocks over which they pass or that are frozen into them : how they undercut and mark the faces of precipices past which they move ; how moraines are formed at the melting ends of them, and so forth. When a district exhibits all the circumstances that are now observed to attend the action of glaciers the proof of the hypothesis that glaciers were once there is complete." (Minto's *Logic*, pp. 349—350.)

(4) Consilience of Inductions.

(4) *A hypothesis must be in harmony not only with facts which it undertakes to explain, but also with facts known in other departments of Nature.* The more a hypothesis tends towards unity, harmony, or consistency, the stronger is the evidence in its favour. Thus, a hypothesis should unite all phenomena of a class which it is required to explain, whether present, past, or future. Prediction, as Whewell mentions, is one of the marks of a valid hypothesis. What is described by Whewell "as the consilience of inductions from different and sepa-

rate classes of facts" is specially important in this connection. There is a greater likelihood of truth when "the hypotheses which were assumed to account for *one class* of facts are found to explain *another class* of a different nature." (*Novum Organon Renovatum*, p. 95.) Thus, the evidence in favour of Gravity was strengthened when the terrestrial form was supported by the celestial. Such coincidence can scarcely be accidental; it points rather to truth.

The proofs of a hypothesis, then, are, briefly, verification, exclusion of rival hypotheses, and agreement with known laws.

Hypothesis is sometimes used in the sense of abstraction.

§ 6. **Hypothesis and Abstraction.** Dugald Stewart and others sometimes employ the term hypothesis to express an abstraction. Geometrical demonstration, for example, is said to be based on hypotheses relating to a mathematical point, line, *etc.*, which do not correspond to realities. It is true, no doubt, that there are certain points of similarity between a hypothesis and an abstraction. Thus, (1) both are representative in character, and (2) in both the actual is explained by the ideal. But though there are these points of community, yet the two differ in important respects. (1) The essence of hypothesis lies in the guess or conjecture with regard to the unknown; but in abstraction no such feature is present. In the latter, we merely turn our attention from the concrete peculiarities of a case and direct our attention to one feature alone, conceived in some cases in an ideal form. Thus, in thinking of a mathematical line, we over-

Points of similarity between hypothesis and abstraction:

(1) Both are representative and (2) both try to explain the actual by the ideal.

Points of difference:

(1) Unlike abstraction, hypothesis implies a guess about the unknown.

look its breadth, and in thinking of a smooth plane, we withdraw our attention from its asperity. (2) Another important point in which hypothesis and abstraction differ is that the consequences following from the former agree with facts, while the results of abstract reasoning are of an ideal character not tallying with facts. The abstract reasonings employed in the different sciences are applicable to facts only when the conclusions are modified by reference to the concrete peculiarities of a case. Thus, the character and functions of hypothesis and abstraction are not quite the same. Hypothesis, no doubt, involves abstraction; but abstraction does not necessarily involve hypothesis. They are connected, but not identical.

(2) Deductions from a valid hypothesis agree, while those from an abstraction do not, with actual facts.

Related to the Method of Abstraction we have what is called the mathematical Method of Limits. A limit may be defined as an extreme case to which all actual cases approach without ever coming up to it; for example, in mathematics, a curve may be considered as the limit of a polygon with its sides increased beyond number. Likewise, an abstraction may be said to be the limit of concrete objects in the sense that it is related to concrete individuals as a curve to many-sided polygons: what is true of the abstraction is true of concrete cases, the more nearly they approach the abstraction. It is said in this connection, 'what is true up to the limit is true at the limit.' The following remark of Whewell is instructive in this connection. "The *Idea of a Limit* supplies a new mode of establishing mathematical truths. Thus,

Method of Limits.

Limit defined.

Abstraction may be regarded as a limit of individuals.

with regard to the length of any portion of a curve, a problem which we have just mentioned ; a curve is not made up of straight lines, and therefore we cannot by means of any of the doctrines of elementary geometry measure the length of any curve. But we may make up a figure nearly resembling any curve by putting together many short lines, just as a polygonal building of very many sides may nearly resemble a circular room. And in order to approach nearer and nearer to a curve, we may make the sides more and more small, more and more numerous. We may then possibly find some mode of measurement, some relation of these small lines to other lines, which is not disturbed by the multiplication of the sides, however far it be carried. And thus we may do what is equivalent to measuring the curve itself ; for by multiplying the sides we may approach more and more closely to the curve till no appreciable difference remains. The curve line is the *Limit* of the polygon ; and in this process we proceed on the *Axiom* that "What is true up to the Limit is true at the Limit." " (*History of Scientific Ideas*, Bk. II, C. 12.)

All real inference is said ultimately to rest on the hypothesis or assumption of the Uniformity of Nature.

§ 7. **Hypothesis and Uniformity of Nature**. In a certain sense, hypothesis may be said to be at the basis of all real knowledge and all science If we exclude formal truth and perfect induction, in which the conclusion is necessarily implicated in the data, we find that all real advance in knowledge or progress in science ultimately rests on the *assumption* of the Uniformity of Nature. The Uniformity of Nature is, no doubt,

regarded by some as an intuitive principle and by others as due to wide and uncontradicted experience. (*Vide* Chap. XVII, § 2 and § 11.) In either case, however, it is viewed as an assumption lending support to every generalization. "All inference," says Jevons, "proceeds upon the assumption that new instances will exactly resemble old ones in all material circumstances; but in natural phenomena this is purely hypothetical, and we may constantly find ourselves in error." (*Elementary Lessons in Logic*, p. 225.) And he writes with regard to our own sphere of existence, "It is a mere assumption that the uniformity of nature involves the unaltered existence of our own globe. There is no kind of catastrophe which is too great or too sudden to be theoretically consistent with the reign of law. For all that our science can tell, human history may be closed in the next instant of time. The world may be dashed to pieces against a wandering star; it may be involved in a nebulous atmosphere of hydrogen to be exploded a second afterwards; it may be scorched up or dissipated into vapour by some great explosion in the sun; there might even be within the globe itself some secret cause of disruption, which only needs time for its manifestation." (*Principles of Science*, p. 748.)

The terms 'certain' and 'probable' are relative, there being degrees of certainty and probability.

Though it is true, from the human stand-point, that absolute certainty with regard to future events is unattainable by man, yet we should not overlook the fact that there are degrees of certainty or probability. What we call 'certain' has a high degree of probability; and what we call

'probable' has but a very weak degree of certainty. And if hitherto no exception has been noticed to the law of uniformity, then for all practical purposes it may be treated as 'certain.' Moreover, it is highly undesirable to call this law a hypothesis, which is ordinarily applicable to our conjectures lying in a tentative stage. "It seems undesirable," observes Mr. Read, 'to call our confidence in Nature's uniformity an 'hypothesis': it is incongruous to use the same term for our tentative conjectures and for our most indispensable beliefs. 'The universal postulate' is a better term for the principle which, in some form or other, every generalization takes for granted." (*Logic*, pp. 264-265.)

It is thus undesirable to call our confidence in Nature's uniformity a mere hypothesis.

Ambiguous use of the term 'Theory':

(1) It sometimes means a hypothesis proved to be true; and

§ 8. Hypothesis, Theory, and Fact. The term Theory has been used by some to indicate a Hypothesis which has been proved to be true in innumerable instances. The line of distinction between Theory, thus understood, and Hypothesis is, however, a delicate one. The proof which the sanguine may consider as adequate to Theory may be considered by the sceptical as rather insufficient; and thus what would be called a Theory by one would be called a hypothesis by another. But, beyond this doubtful application of Theory, there is a sense which is generally accepted: Theory is understood to imply a system of truths or laws relating to a particular subject-matter, constituting the province of a definite department of knowledge or science (*e. g.*, the atomic theory, theory of dew, theories of light and heat).

(2) sometimes it means a system of truths constituting a science.

Difference between Fact and Theory :

(1) Sometimes 'Fact' means what is immediately present before us, while 'Theory' implies the truths established by inference.

The term 'Fact' comes from Latin '*factum*,' implying what has been done. 'Fact' thus signifies what has been done or what has happened. 'Fact,' accordingly, covers all those presentations which are immediately known and whose truth, therefore, can never be questioned. (*Vide* Chap. I, § 1 and § 9.) Theory, on the other hand, comes within the province of inferential knowledge—what is legitimately thought out to explain a class of facts.

(2) Hence 'Fact' is supposed to refer to the concrete and individual, while 'Theory' to the general and abstract.

Again, as intuitive knowledge is restricted to the concrete and individual, 'facts' are supposed to be essentially concrete and individual ; and, as inference has to do mainly with general truth, we find theory is often used in the sense of a system of such truths.

(3) Sometimes 'Fact' is used in the sense of what is objectively certain.

In some cases, however, the term 'Fact' is applied to a law or general truth, when it is proved beyond dispute (*e.g.*, when we speak of the 'fact' of gravitation or of the mortality of man). In such cases the term 'Fact' is extended to cover general truths simply because these approach the certainty of individual instances or concrete cases. 'Fact' in this sense means what is objectively true or certain.

In the last sense the terms become relative : a lower generality is regarded as a 'Fact' in relation to a higher, which is viewed as a 'Theory'. Dr. Whewell.

Whewell observes in this connection, "At any one of the steps of Induction, the Inductive proposition is a *Theory* with regard to the Facts which it includes, while it is to be looked upon as a *Fact* with respect to the higher generalizations in which it is included. In any other sense, the opposition of *Fact* and *Theory* is untenable and leads to endless perplexity and debate. Is it a Fact or a Theory that the planet Mars revolves in an Ellipse about the Sun ? To

Kepler, employed in endeavouring to combine the separate observations by the conception of an Ellipse, it is a Theory; to Newton engaged in inferring the law of force from a knowledge of the elliptical motion, it is a Fact. There are no special attributes of Theory and Fact which distinguish them from one another. Facts are phenomena apprehended by the aid of conceptions and mental acts, as Theories also are. We commonly call our observations *Facts*, when we apply, without effort or consciousness, conceptions perfectly familiar to us: while we speak of Theories, when we have previously contemplated the Facts and the connecting Conception separately, and have made the connection by a conscious mental act. The real difference is a difference of relation; as the same proposition in a demonstration is the *premise* of one syllogism and the *conclusion* in another;—as the same person is a father and a son." (*Novum Organon Renovatum*, p. 116.) Thus, there is no fundamental separation between facts and theories. What we call facts are but results of previous investigation and theorizing as well as the starting-point for fresh inquiry and hypotheses. There is, accordingly, a continuity between facts and theories, they being but different stages of the Inductive process: theories when proved and generally accepted coming to be known as facts, and facts when carefully examined revealing their theoretical basis. This relative difference is, therefore, due to the circumstance that a lower generality, in relation to a higher, is viewed as a

to the abstract interests of communities in general is to employ this method, as the conclusions are then based, not on an actual estimate of facts in any particular case, but on a theoretic estimate of abstract conditions.

Though ordinarily the above forms of the Deductive Method have special spheres of their own, yet they may be extended to the other spheres as well.

Though generally the above forms of the Deductive Method are restricted in their sphere of inquiry, yet we find that, as a matter of fact, no such restriction is possible. The Direct Method, though generally restricted to the explanation of physical phenomena, may sometimes be applied to the study of historical facts; and the Inverse Method may sometimes be applied to the study of physical phenomena. We find, for example, that the laws of Keplar were first determined empirically before they were deduced from the universal law of gravitation. Here, then, observation and generalization preceded the application of the Deductive Method. We likewise find that the Geometrical Method also is sometimes applied to phenomena other than those relating to space and time. We find, for example, the tendency among some politicians simply to deduce consequences from abstract political principles and not to appeal to experience for verification. The opposite tendency is sometimes found when politicians are content only with observation and empirical generalizations, which are not confirmed by deductive calculation.

A fact is conclusively established when Induction is

§ 4. **Induction Aided by Deduction.** The full force of scientific method, as Bain observes, is illustrated when Deduction is brought

supported by Deduction.

to the aid of Induction. To conclusively establish a fact, we should not be satisfied merely with Inductive generalizations from experience, nor with the mere abstract calculation of result by the Deductive Method. The former course is precarious, as it is purely empirical ; and the latter is no less uncertain (so far as the actual result is concerned), as it is purely theoretical. To establish a fact or law beyond dispute we must supplement Inductive inquiry by Deductive calculation. We may illustrate our remark by reference to an example cited by Bain. It is found, for example, that the N. E. wind is generally injurious to health. It is first generalized empirically by observation : whenever the N. E. wind blows, the health suffers. If the same effect can be accounted for by calculation, then the generalization would be put on a firmer footing. It is found that the N. E. wind blows in contact with the ground and is often charged with impurities and noxious elements. If we take these factors into account and deduce a result therefrom, we find that the wind cannot but be injurious to health, since the impurities and noxious elements tend to undermine it. Thus, a prior Induction is subsequently confirmed by Deduction, and the matter is, so to speak, proved beyond doubt.

An example.

§ 5. **Exercises.**

1. By what form of reasoning is it possible to ascertain beforehand the effects of composite causes ? In what sciences, and in what professions, is reasoning of this kind most essential ?

2. Indicate the different forms of Deductive Reasoning. Explain and illustrate the importance of what Mill calls 'the Deductive Method of Induction.' Does he himself realize its full importance?

3. Determine the scope of the Deductive Method in Induction. How is such a method related to Hypotheses?

4. Can the provinces of Induction and Deduction be kept separate? Elucidate—"Combined Induction and Deduction expresses the full force of scientific method for resolving the greatest complications."

5. When is the Deductive Method employed in inductive investigation? Point out the conditions of the right use of this Method. Is verification always necessary?

6. Distinguish between the Direct and the Inverse Deductive Method. Why have the methods been called the Physical and the Historical Method respectively? Are there valid grounds for such names?

7. What is the Geometrical Method? Why is it so called? How is it related to the other Deductive Methods?

CHAPTER XXI. → omitted whole

THEORY OF PROBABILITY OR ELIMINATION OF CHANCE.

As a plurality of causes frustrates most of the inductive canons, we require some other procedure which may be an aid to generalization in such a case.

§ 1. Necessity of Probability in Induction. The second difficulty in applying the Inductive Canons is felt in the case of plurality of causes. Without examining here the validity of the hypothesis of the plurality of causes, we may assume it as it is advocated by Mill and Bain, specially as, owing to our imperfect observation or knowledge, it is not always possible for us to discover exact correspondence between cause and effect. If one and the same effect be apparently produced sometimes by one cause and sometimes by another, then it is hard to determine, by the employment of the Inductive Canons, the causal link in such a case. In such cases, we can, however, determine with a certain degree of exactitude the causal connection by the employment of the Theory of Probability or Chance. If, for example, an effect X may be produced by either A or B, then we try to determine by the doctrine of chance whether the presumption in favour of A is greater than that in favour of B : we calculate by the doctrine of chance how often A and X are likely to go together, if due only to accident. If we find the connection to be more frequent, then we naturally suppose that the relation is not merely *casual* but that there is a *causal* link between the two. (*Vide* § 5.) We should, accordingly, confine our attention in this

The theory of probability succeeds in such a case.

It enables us to detect a causal connection by the elimination of chance-coincidences.

Chapter to the study of the Theory of Probability, so far as it bears on the inductive determination of causes.

The scientific conception of the universe really leaves very little room for accident or chance. Properly speaking, every phenomenon or occurrence is determined by its cause or conditions, proximate and remote, known and unknown. As, however, there are limits to human knowledge, we cannot expect to discover or to know the causes of all phenomena or coincidences. Thus, there is room for chance or probability. In fact, Chance is taken by Mill as implying a coincidence which does not afford any ground for establishing uniformity.

Owing to the imperfections of our faculties, events seem to us to be accidental or probable.

Mill's definition of Chance.

Probability has been used in two different senses : (1) popularly it implies a greater likelihood for than against an event ; (2) scientifically, however, it refers to any situation below certainty and above impossibility. Probability has to do with approximate generalizations ; and, when such generalizations are reduced to statistics, the calculation of probability becomes more definite—at least with regard to the average number of cases. No doubt, probability has also been applied to individual instances, but with no greater certainty. (*Vide* § 3.) As Bain remarks, "When a sufficiently extended series of observations shows a fixed proportion in the relative occurrence of events, this proportion is called the probability of the occurrence of any single event : which, however, is a fiction meaning only the certainty of the proportion, or average, on the whole." (*Induction*, p. 91.)

Ambiguous use of the term 'Probability'.

It is sometimes maintained that all inference ultimately rests on probability;

§ 2. Inference, Chance, and Probability. "All inference," says Jevons, "is relative and hypothetical." (*Elementary Lessons in Logic*, p. 225.) The remark is, no doubt, true to a certain extent. We have seen that induction proper, involving a reference to the unknown—the distant and the future,—is more or less uncertain or hypothetical. (*Vide* Chap. XIX, § 6.) If, for example, Nature be not always found to be uniform in all respects, then what is taken to be true here and now may not be true elsewhere and hereafter. Similarly, the material validity of a syllogistic conclusion depends on that of the premises. If the premises be uncertain, the conclusion must be so too. If, therefore, all universal propositions be uncertain, the conclusion must always be probable; and, in this sense, the dictum of Bishop Butler that 'probability is the very guide of life' is true. From this stand-point we have to do with probability in every case; the question in any particular instance being to determine its degree. If we adopt this position, then the terms 'sure' and 'certain,' and all analogous expressions, will have to be erased from language. A little reflection shows, however, that when we use these expressions we mean only such certainty as we, with our imperfect faculties, can possibly attain, and not the degree of certainty which an Absolute Being alone can possess. Thus, truths which are based on the laws of thought and countenanced by the unvarying order of things are taken by us as certain, and not as probable. If these justify universal propositions, we ac-

but this view is not tenable.

Generalizations justified by the laws of thought and the uniformity of nature are to be treated as certain.

cept them as true. When, on the other hand, facts do not warrant universal propositions, we are in doubt about our inference in any case, and the degree of doubt is determined by the balance of evidence furnished by facts.

Universal propositions ensure certainty of conclusion, while particular propositions give rise to doubt.

It is this doubt which generally lies at the bottom of what are called approximate generalizations or particular propositions and gives rise to probability. As, however, a condition of doubt or uncertainty is one of perplexity, we often try to avoid it by aiming at universal truths yielding certain conclusions. And so we try to render even approximate generalizations precise that they may, within a certain sphere, be treated as universal propositions, yielding definite results. We try to attain our end in two principal ways :—

As doubt is painful, we try to render particular propositions definite by discovering the sphere within which they may be treated as universal.

This is sometimes effected in two ways

(1) We may ascertain exactly the exceptions to a rule, which is viewed as indefinite because the exceptions are not known. Thus, instead of being content with the indefinite truth 'Some metals are solid' (which justifies only a probable conclusion in any case), we determine precisely that 'All metals except mercury are solid,' which justifies a certain conclusion on any occasion. It is in this way that the rules of Grammar and the laws of a State are made general by a separate specification of their exceptions.

(1) by enumerating the exceptions,

(2) Generalizations, which seem to be approximate, owing to our ignorance of the exact circumstances in which they are always true, may turn out to be universal when these circumstances are discovered and mentioned as defining the precise

and (2) by determining precisely the circumstances in which such propositions are always true.

sphere of their application. Thus, instead of being satisfied with the vague proposition 'Some kinds of fever are cured by quinine' (which is often practically useless), we try to be more precise and say 'Chronic ague is cured by quinine,' which is of great practical value. Similarly, instead of holding 'absolute monarchs generally abuse their power,' we maintain that 'An absolute sovereign will abuse his power, *unless* his position makes him dependent on the good opinion of his subjects, or *unless* he is exceptionally good and firm or is guided by ministers who are so.'

When, however, we fail to universalize a proposition, we are content with probability supported by statistics or balance of evidence furnished by facts.

When, however, we fail to reach a universal proposition in any way, we are content with as much certainty as attainable; and we try by means of statistics to arrive at an objective estimate of probability. Probability, as we have said, is connected with doubt or uncertainty. And, as such uncertainty is due rather to our estimate of facts than to the facts themselves, probability has been regarded by some as purely subjective, without any objective validity. Though, it is urged, all natural events may be the outcome of necessity, yet our ignorance of the conditions of their production gives them an aspect of contingency.* It may be said, however, that contingency or uncertainty rests not merely on the degree of subjective conviction but also on the imperfect estimate of facts: so long as all the circumstances are not taken into account, the few, which are considered,

Probability is thus not purely subjective, but supported by evidence.

* The student may consult in this connection *The Elements of Morals*, Chap. XX, § 8.

justify only an approximate generalization.* And the more we have access into the secrets of Nature, the more correctly do we discover the numerous conditions or circumstances yielding a definite result. Herein we find the distinction between Chance and Probability. Chance is simply due to our ignorance; and, so long as we do not know the circumstances which bring about a phenomenon, we attribute it to mere chance or accident.† Thus, we attribute the success of an enterprise to chance when its conditions are not known, as, in olden times, an eclipse of the sun or moon was deemed accidental owing to an imperfect knowledge

Distinction between Chance and Probability.

* Even if the 'few' circumstances, which are taken into account, be the same always, they would yield one uniform result; but, as a matter of fact, they are taken in varying combinations and thus they give rise to variable results, with some preponderance in favour of one owing to the predominant influence of some of the factors present in all the cases. For example, when we try to ascertain the probable effect of education on individuals, education itself involves various factors and diverse forms and individuals likewise have multifarious traits, due to native endowment, heredity, environment, etc. And these factors taken in different combinations would no doubt give rise to different results, though there may be some general agreement in many cases owing to the influence of some common factors affording room for the calculation of probability. It is, therefore, the study of mixed cases which lends support to the doctrine of probability, while an examination of pure cases or exactly the same combination of factors would justify only an inductive generalization.

† Jevons writes—"Chance cannot be the subject of the theory, because there is really no such thing as chance, regarded as producing and governing events. The word chance signifies *falling*, and the notion of falling is continually used as a simile to express uncertainty, because we can seldom predict how a die, a coin, or a leaf will fall, or when a bullet will hit the mark. But everyone sees, after a little reflection, that it is in our knowledge the deficiency lies, not in the certainty of nature's laws. There is no doubt in lightning as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach, but infinite knowledge would account for its lying there; and the course of every falling leaf is guided by the principles of mechanics which rule the motions of the heavenly bodies". (*Principles of Science*, p. 198.)

of planetary motions. As, however, our knowledge of circumstances advances, chance is gradually eliminated, affording definite data for the calculation of probability. Chance rests on ignorance, while probability on imperfect experience. Hence, everything is possible when referred to chance, though the probability in its favour may be but little.

§ 3. The Average and the Personal Equation. We have said that Probability is due to imperfect experience and is connected with doubt. Doubt is a wavering condition of the mind lying between Belief and Disbelief, both of which imply perfect assurance either of the presence or of the absence of a fact. But, doubt, to be legitimate, must be based on conflicting or inadequate experience and not merely on idle speculation. And only such reasonable doubt is connected with probability. From this it follows that whatever conflicts with well-established inductions or verified laws of Nature should be disbelieved and treated as untrue or even as impossible.* But, within the extreme limits of Belief and Disbelief, there is the region of

Reasonable doubt underlies p obability :

* We may consider here three grades of impossibility: (1) Whatever contradicts the laws of thought or mathematical truths is to be treated as absurd. If any one says that John is both dead and alive, or that an effect is produced without a cause, or that two and two in any case are found to be five, we are to treat such an assertion as false. The following case relating to circumstantial evidence also illustrates the point: "A person having been robbed and murdered, the body is so placed by the offender, with a discharged pistol beside it, as naturally to induce the inference that the deceased had fallen by his own hand: but, on close examination, it is discovered that the ball extracted from the body, and which occasioned death, is too large to have been discharged from that pistol, an inconsistency which immediately

Doubt which must be enlightened by experience. We should remember that we are concerned here with doubt, only so far as it is connected with inference, as determined by particular propositions or approximate generalizations. And we must be guided here by any preponderance of evidence furnished by facts. When statistics are available, we are guided by them, as they are definite and so afford a reasonable ground of expectation. In the absence of statistics, we are to weigh subjectively the circumstances for and against any particular conclusion before arriving at any result. And, in this subjective estimate, we do not overlook even minor factors, since they contribute their share towards either side of a question. "In probability," says Venn, "we distinctly take notice of, and regard as evidence, reasons so faint that they would scarcely be called by any other

it is connected with inference determined by particular propositions.

We should be guided here by the balance of evidence furnished by facts (*e.g.*, statistics).

In the subjective estimate of probability, even minute

detects the imposture, and refutes the false inference to which *some* of the circumstances apparently tend." [Norton's *Law of Evidence*, p. 159.] (2) Whatever contradicts the physical laws, such as those of gravitation, combustion, birth, and sustenance, is to be regarded as false. That an unsupported body remains suspended in air, that fire does not burn, that a man spontaneously comes into existence without being born of parents, or that a man lives without food for an indefinite length of time is a position which is altogether inconsistent with physical laws. The onus of proof in such cases lies with the persons who put them forth. (3) Whatever contradicts the laws of human nature (such as the laws of attention, memory, or affection) is to be viewed as untrue. If any one maintains that concentration of attention on a topic renders it indistinct or that he remembers something without any suggestive force, his declaration should not be accepted as true. A good illustration of this sort of incredibility is found in the celebrated judgment of Solomon when he had to decide between the rival claims of two women for one and the same child, alleged by each to be her own. When he directed that the child be divided into two and a part be given to each, then the natural feelings of the true mother led her to request that the child be spared though she might not get any part. (1 Kings, III, 16—27.)

and minor details are noticed.

name than mere hypothesis elsewhere." (*Logic of Chance*, p. 127.) The computation of probability occupies a very prominent place in the logic of life, as most of its issues are determined by such computation. Whether, for example, a nation or party, engaged in conflict, would be victorious or not, whether an individual should adopt this or that profession, whether a patient suffering from a disease would recover or die, whether an alliance would be advantageous or injurious can be settled only by an estimate of probabilities. "The whole life of man," observes Buckminister, "is a perpetual comparison of evidence and balancing of probabilities."

The complex affairs of life often require the use of the principles of probability.

The conclusions of probability are true only of the average number of cases.

It should be borne in mind in this connection that a computation of probability is true only with regard to the average number of instances, individual cases being always more or less uncertain. When we study a large number of instances of any phenomenon, say a disease like cholera, we find that a certain proportion of cases is fatal; and the more extended our observation, the less does the proportion fluctuate. This proportion indicates the *average* number of cases true on the whole. Though this proportion is also employed to determine the probability of a single event, yet, as a matter of fact, it is as uncertain as ever. When, for example, knowing the average proportion of fatal to non-fatal cases of cholera to be, say, 4 : 1, we say with regard to a particular patient, suffering from cholera, that the chance of his recovery is 1 : 4, we evidently employ a fiction, for the chance of his recovery is as uncertain as ever, whether we

judge the case by the proportion or not. All that we can say is that, in the long run, the average number of recoveries in the case of cholera is indicated by the proportion 1 : 4. "Unless we act upon the gross or total, we gain nothing by taking into account the numerical probabilities with a view to a single instance." (Bain, *Induction*, p. 91.)

The reason of this is found in the fact that the central law, governing a class of complex objects or events, stands out prominently when a large number of instances is observed.

The law is illustrated in most of them, though it is frustrated in some owing to counteracting agencies.

The explanation of the above fact is found in what has been said above about the ultimate grounds of probability. We have mentioned that, in this well-ordered and harmonious world, everything is governed by law; so that what we call accident is merely due to our ignorance. In every individual case, an effect is produced by the convergence or conflict of numerous laws, which it is not easy to determine. Hence we are often perplexed in our estimate of individual instances which seem to baffle inquiry and explanation. When, however, we study a large number of instances of a class, the law governing its characteristics stands out prominently in the midst of individual peculiarities. And the wider our observation, the greater the chance of detection of such a law. Hence the value of average in any sphere. The cholera poison or germ, for example, is governed by a law of its own, which is injurious to the human organism. But, in any particular case, there may be the operation of other laws (such as those connected with habit, diet, surroundings, medicine, inherited constitution) counteracting such influence. When, however, we study a very large number of cases, these counteracting influences are, more or less, excluded by

Thus, the calculation of probability illustrates also the employment of the Inductive Methods with the connected processes of varying the circumstances and elimination.

the Method of Agreement, and the central law concerning the cholera poison may approximately be read in what is known as the average. Thus, the calculation of probability illustrates also the application of the Inductive Methods—but only to the examination of very complex cases, rendered difficult by the intricacy of relation subsisting among the several laws governing them. The processes of varying the circumstances and elimination are illustrated in the study of a very large number of cases and the exclusion of peculiarities or (as they are called in this connection) deviations from the central law governing the type.*

Mr. Read's account.

"The calculation of probabilities," says Mr. Read, "supposes a class or series of instances or events subject (as far as known) to somewhat similar conditions, though the conditions are not so similar

* Bain writes, "So far as the mere decay of the human system is concerned, deaths would be equally frequent at all periods of the year, and at all hours of the day. In the statistics of Mortality, however, we find that some months are marked by an excessive number of deaths; as December, January, and February. This points to a law of connexion between winter severity and mortality. In the same way, if we had the statistics of the deaths occurring at different hours of the day, we might find a greater number occurring in the depressing hours of the night, namely, between midnight and dawn. There is an element of chance, and an element of law; the chance can be eliminated by statistics, and the law ascertained and estimated." (*Induction*, p. 89.) Statistics in these cases merely reveal the influence of the weather or atmosphere on the human organism, as distinguished from other circumstances. Here is another analogous instance:—"Apoplexy occurs more often in males than in females and frequently between the ages of fifty and seventy than in the other periods of life. With respect to age, the disease is rare before twenty-two years, and increases in frequency with the increase of age from twenty-two years upwards. It occurs most often during the cold season of the year, and according to the observations of Sarmani the hours from three to five o'clock in the afternoon, and two to four in the morning, are those in which the greatest number of cases occur." (*The Home Hand-book of Domestic Hygiene and Rational Medicine* by Dr. J. H. Kellogg, p. 1080.)

as to result in uniformity. Where the more similar conditions predominate, they produce average instances; where dissimilar conditions occur, but in such a way as to cancel one another, the average again results; where unusual conditions occur without cancelling, extraordinary instances appear. Hence, if the average height of a nation is 5 ft. 6 in., most men will be about that size; men of 5 ft. and 6 ft. will be rare, and those of 4 ft. 6 in. and 6 ft. 6 in. rarer still; whilst limits to height in both directions seem to be fixed by the nature of things." (*Logic*, p. 293.) These 'deviations' or 'errors' (as they have sometimes been called) are evidently less numerous as we come to extreme cases, since a special combination of circumstances, prominently interfering with the operation of the central law governing a type, is comparatively rare indeed.

Personal Equation indicates the average tendency in an indivdual to depart from the normal estimate of a class of facts.

The question of 'deviations' or 'errors' brings us to what is known as the *Personal Equation.* Every individual has in him a common nature belonging to the other members of his species, and also certain features, which are peculiar to himself. Among these features, again, we find some which are comparatively constant, such as the influence of personal character or predominant tendency, and some which are highly variable, such as the influence of passing desires, inclinations, and dispositions. Now, in the case of calculations on common data, we naturally expect agreement among different individuals owing to the presence of a common rational nature in all of them. But

It is due to the influence of personal character.

there are sources also of variation owing to personal differences. Among such differences, we find certain fixed tendencies in different individuals which are the expression of personal character. Thus, some may be generally disposed to make mistakes about dates; some, about places; and some, about names. This influence of personal character which gives a bias to one's calculations in a certain direction is known as his personal equation. "Each Man," writes Mr. Read, "has a certain cast of mind, character, physique, giving a distinctive turn to all his actions even when he tries to be normal. In every employment this determines his Personal Equation, or average deviation from the normal. The term Personal Equation is used chiefly in connection with scientific observation, as in Astronomy. Each observer is liable to be a little wrong, and this error has to be allowed for and his observations corrected accordingly."* (*Logic*, pp. 294-295).

Statistics often suggest laws.

§ 4. **Importance of Statistics**. From the above account it is clear that statistics are of material help in settling questions of probability and in determining also the character of average in any case. When we cannot directly discover

* The term 'personal equation' was first used in Astronomy. In 1795 the presence of an error in the observations of transits was detected in the case of a Greenwich observer, who was dismissed in consequence. Its existence as a general feature of the work of all observers was noticed by Bessel and others in the beginning of the last century. Personal equation thus indicates "An error made by a person in a measurement or exact observation of any kind, which is peculiar to himself, and which must therefore be determined and allowed for when the precise result of the observation or measurement is to be derived." (Professor Newcomb.)

the causal link in complex cases, we have to surmise it by weighing evidence gathered from a careful examination of facts. And if all facts of a class do not reveal a uniform relation or law, discoverable by inductive generalization, we are constrained to study a good number of them in a methodical way with the hope of discovering some obscure relation which would otherwise escape our notice. Herein lies the place of statistics*' which systematically collect facts relating to a field of inquiry. "In proportion," writes Sigwart, "as we are unable to reduce the particular event to rules and laws, the numeration of particular objects becomes the only means of obtaining comprehensive propositions about that which is, for our knowledge, fortuitous; as soon as the laws are found, statistical numeration ceases to be of interest. There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur, and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunder-storms and hail-storms occur at a given place, or within a given district, how many persons die, and how many bushels of fruit a given area produces, because we are not in a position to calculate these events from their conditions." (*Logic*, Eng. Trans., II, p. 483.)

Statistics methodically collect facts by counting in a field of inquiry.

* Statistics have been defined by Rümelin as "the results obtained in any field of reality by methods of counting."

Statistics are useful in complex cases which do not directly reveal their laws.

Statistics, then, are employed (1) when the facts we are studying are complex and (2) when these do not directly reveal a general relation or law governing them. But, in order that statistics may be of any use, there should be methodical, and not mere random, enumeration. As in an inductive inquiry our observation and experiment must be regulated by a guiding hypothesis in order that they may help us in the discovery of a causal link, so in statistical enumeration we must sub-divide the field of our inquiry into as many groups as possible quite in a logical or systematic way and then by comparing the statistics of the different groups we should try to gather any causal relation lurking in them.

Statistical enumeration, to be useful, must be systematic and comparative by reference to the sub-divisions of the class of facts we are studying.

Illustrations.

Thus, to determine the longevity of a particular race or nation we may gather the statistics of its members grouped under certain heads—such as age, sex, habits, education, profession, *etc.*—so that by eliminating the peculiarities found in particular sections we may arrive at a fair estimate of the average. Similarly, to determine the havoc wrought by malaria, we gather from various districts the statistics of deaths and sufferings of people varying in constitution, customs, habits, tastes, *etc.* Thus, statistical investigation, to be fruitful, should be well-regulated and not diffused or aimless.

The employment of the comparative method, tends to the discovery of laws.

Methodical and classified enumeration by reference to sub-divisions affords room for the employment of the comparative method, which is more likely to suggest a causal link than the mere random numerical study of a whole class in its vague and indefinite generality.

Induction

Both in Induc-

tion and Statistics our observation, analysis and comparison should be guided by a hypothesis in order that these may be means of real discovery.

and statistics should be regulated by a hypothesis.

The advantages of statistical enumeration are briefly the following :—(1) It gives us a comparatively exact and comprehensive grasp of facts, as distinguished from the vague and narrow experiences of ordinary life. (2) It thus enables us to discover a causal connection suggested by quantitative correspondence (*e. g.*, the number of births or deaths and the favourable or unfavourable condition of climate, weather, surroundings, or provisions), ignorance of which would obscure the relation. We should remember here that statistical uniformities afford a basis only for empirical generalizations. [*Vide* Chap. XXIII, § 3 and § 4.] (3) It, accordingly, helps us in forming an idea of the average, which furnishes a ground for probable conclusions in the future, as explained in the last section.

Advantages of statistics :
(1) They enable us to have a precise and comprehensive estimate of facts.
(2) They suggest empirical laws.
(3) They help us in forming an idea of the average.

Owing to these advantages, statistics are extensively employed in various departments in modern times,—specially as it is often difficult directly to arrive at inductive generalizations owing to the complexity of materials in many cases. A cue furnished by statistical correspondence usually supplies a ground for subsequent inductive investigations, leading at times to unforeseen discoveries. And here we find an important difference between ancient and modern science : "The extent to which the statistical method prevails, and everything is counted," ob-

Modern science generally makes an extensive use of statistics.

serves Professor Sigwart, "is another instance of the fundamental difference between ancient and modern science." (*Logic*, I, p. 286.)

Both Induction and Probability rest on observation or experience: but while the former has to do with universal propositions, based on uniform experience, the latter has to do with particular propositions, based on variable experience.

§ 5. Probability and Induction. Probability implies, as we have seen, (1) subjective conviction and (2) experience giving rise to it. That probability may be of any service to Inductive Logic, it must be based on observation or experience. In fact, probable generalizations are also inductively arrived at. The difference between Induction and Probability lies in the fact that, though based on observation or experience, the former has to do with universal propositions or generalizations without any exception, while the latter has to do with particular propositions or approximate generalizations. It has been said by some that Induction itself is based on Probability. When, for example, it is said that there is the succession of day and night it is not an absolute certainty. The conditions of the universe are numerous and inscrutable; and if some of these change, the so-called Inductive generalizations, which are based on the existing order of things, will also be affected. Thus, there may be, under changed circumstances, a perpetual day or a perpetual night.

It is sometimes contended that Induction, being more or less uncertain, is itself based on Probability.

But this view is not tenable. It rests on an ambiguous use of the term 'certainty.'

If we examine this position with a little care, we find that it is based on an ambiguous meaning of 'certainty.' Certainty may mean (1) absolute certainty or (2) rational certainty as justified by a comprehensive view of the existing order of things. (1) If probability means less than absolute certainty,

then, no doubt, everything is probable; and induction also is so. But (2) if probability means less than rational certainty, then inductive generalizations can never be said to be probable, for they are certain within reasonable limits. In fact, Probability, as we have seen, rests on Induction; and not Induction, on Probability. (*Vide* § 3.) As Mr. Bosanquet says, "Ultimately the calculus of chances may be said to rest on the same principle as Induction, in so far as the repetition of examples derives its force from the (unspecified) variety of contexts through which this repetition shows a certain result to be persistent. But in such a calculus the presumption from recurrence in such a variety of contexts is only estimated, and not analysed." (*Essentials of Logic*, foot-note, p. 144.)

Probability rather rests on Induction.

§ **6. Logical Grounds of Probability.** The Inductive evidence of Probability may rest (1) either on direct statistics, or (2) on computation determining an effect by reference to the laws of the causes operating for and against its production, when they are known, or (3) on a combination of these two methods, implying a verification of calculation by statistics. Of these three methods the third is comparatively the most certain, and the first is the most precarious. Direct statistics can give but an empirical law, the application of which to definite instances yields highly uncertain results. (*Vide* Chap. XXIII, § 5 and Chap. XXX, § 6.) The calculation of Probability leads to the consideration of Average: Average instances, as

The inductive evidence of probability rests either (1) on statistics, or (2) on computation, or (3) on both.

Of these three methods, the last is the most secure.

we have seen, conform more to the rule of Probability, while extremes deviate from it. It should be remembered in this connection that Probability can give rise to subjective assurance, without any corresponding objective certainty, at least so far as individual instances are concerned. (*Vide* § 3.)

The principle for detecting a causal connection by Probability.

Causal connection is estimated in Probability by the following principle—*Determine the positive frequency of the phenomena themselves, and how great frequency of coincidence must follow from that, on the assumption that there is neither connection nor repugnance. If there be a greater frequency, there is connection; if a less, repugnance.* If, for example, we imagine a person as able to take his walk by the river-side at any time between 6 A. M. and 6 P. M., and (*a*) if we observe him taking a walk at any particular hour once in every twelve days, we naturally attribute the connection to chance. (*b*) If on the other hand, we find him taking a walk at 6 P. M. once in three or six days, instead of once in twelve, we suspect that there is a causal connection between that hour and his walk: in other words, we infer that the person prefers taking a walk at that time. (*c*) If, again, we find the person taking a walk at 12 (mid-day) once in 30 days, instead of once in 12, we naturally conclude that there is a repugnance: in other words, we suspect that the person is disinclined to take his walk at that time. We suspect a causal link here, but it works in the opposite direction as a deterrent. Let us now consider the rules of probability which justify definite conclusions in

An illustration.

different cases. Let us consider them in distinct sections.

The Rules of Probability:

Rule for determining the probability of the concurrence of two independent events.

§ 7. **Rule for Estimating the Concurrence of Two Independent Events.** *If two events are quite independent, having neither connection nor repugnance, the probability of their concurrence is measured by the product of the fractions representing their separate probabilities.* If A occurs once in 5 times and B once in 4, then the probability of their coincidence is measured by the product of the two fractions $\frac{1}{5}$ and $\frac{1}{4}$, *viz.*, $\frac{1}{20}$. The rule may be explained thus :—If we suppose A and B to be both constant, then they are always found together. If, however, A be constant while B is found only once in two cases, then the probability of their concurrence is reduced to $\frac{1}{2}$. If, again, B happens once in three cases or once in four (A remaining constant), then the probability of their coincidence becomes $\frac{1}{3}$ or $\frac{1}{4}$. If we now suppose A also varying, then the chance of coincidence is further diminished. If, for example, A happens once in three cases, while B once in two, then the probability of their coincidence would be $\frac{1}{3} \times \frac{1}{2}$, *i.e.*, $\frac{1}{6}$. Similarly, if A occurs once in 5 times and B once in 4, the probability of their concurrence is measured by $\frac{1}{5} \times \frac{1}{4}$, *i. e.*, $\frac{1}{20}$. The rule may be illustrated by concrete examples. Suppose I am always in my room, and another gentleman is also constantly there. Then, evidently, both of us are always together. But, if the gentleman goes out and comes in every other hour alternately, then we shall be together only half the previous

Illustrations.

number of times. If the previous number (indicating our always being together) be represented by the integer 1, then the subsequent number should be represented by the fraction $\frac{1}{2}$. Suppose, again, that I also, instead of being always in the room, go out and come in every third hour, then the chance of our being together would still more be diminished to the extent of one-third of the preceding number. In other words, the probability in that case would come down to $\frac{1}{3}$ of $\frac{1}{2}$, *i.e.*, $\frac{1}{6}$. If, similarly, it be known that 2 men in every 5 are diligent and 3 men in every 7 are benevolent, then the probability of a man being both diligent and benevolent is $\frac{2}{5} \times \frac{3}{7}$, *i.e.*, $\frac{6}{35}$; *i.e.*, out of 35 men, 6 persons are likely to be both diligent and benevolent. If, likewise, 4 days in 5 are bright and 3 days in 8 are dry, then the probability of their coincidence is $\frac{4}{5} \times \frac{3}{8}$, *i. e.*, $\frac{3}{10}$. In other words, out of ten days, 3 days are likely to be both bright and dry.

Rule for determining the probability of the occurrence of one or other of two events that cannot concur.

§ 8. Rule for Determining the Occurrence of Either of Two Inconsistent Events. *If two events cannot possibly concur, the probability of the occurrence of either of them is measured by the sum of the separate probabilities.* If, out of every 5 mangoes, 2 are sweet, and out of every 7, 3 are sour, then, out of 35 mangoes, there are altogether 29 which are either sweet or sour. This is clear from the following calculation :—Out of 35 mangoes, the number which is sweet is 14 (*i.e.*, $\frac{2}{5}$ of 35), and the number which is sour is 15 (*i. e.*, $\frac{3}{7}$ of 35).

Illustrations.

Thus, altogether there are 14+15, *i.e.*, 29 mangoes which are either sweet or sour; and this is determined by adding together the two fractions representing the separate probabilities (*viz.*, $\frac{2}{5}+\frac{3}{7}$ or $\frac{29}{35}$). Similarly, "If one man in ten is over six feet, and one in twelve under five; then in a large number, say 120,000, there will be about 12,000 over-six-feet men, and about 10,000 under-five-feet men; the sum of the two 22,000, will represent the number of such as are one kind or the other." (Bain, *Induction*, p. 93.) If, likewise, a coin when tossed up presents the head and the tail uppermost equal number of times, then the probability of each is $\frac{1}{2}$. The number of cases, therefore, in which either the head or the tail is presented is the total number of throws. In these cases it is evident that a mango cannot be both sweet and sour, an individual cannot at once be over six feet and under five, nor can a coin simultaneously present both the head and the tail.

Rule for determining the deterioration of testimony as it passes from one person to another.

§ 9. Rule for Estimating Deterioration of Testimony. *When testimony passes from one person to another, it is weakened; and the value of testimony in such a case is determined by the product of the fractions representing the separate probabilities.* For want of proper comprehension, defects of memory, tendency to exaggeration, or mendacity, incorrect accounts of events are at times given by individuals noticing them. When, however, events are not directly observed, but are learnt at second-hand and then reported, the chances of error are increased; and the more

numerous the persons through whom a testimony passes, the weaker does it become. Hence what is called 'hearsay evidence'—*i.e.*, evidence based not on direct observation, but on indirect testimony or what has been heard—is not admissible in courts of law. When B, for

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example, reports what he has learnt from A, the value of testimony evidently suffers. If the veracity of A is measured by $\frac{4}{5}$ (*i.e.*, if he speaks truth four times in five) and that of B by $\frac{2}{3}$ (*i.e.*, if he speaks truth twice in three cases), then the value of B's testimony is determined by the product of the two fractions $\frac{4}{5}$ and $\frac{2}{3}$, *i.e.*, by $\frac{8}{15}$. This can easily be explained thus:—Out of 15 instances, A speaks truth 12 times (*viz.*, $\frac{4}{5}$ of 15). Three cases out of 15 are thus misrepresented by A and so necessarily reported amiss by B. Of the 12 instances of truth learnt from A by B, he would be correct only in 8, *i.e.*, two-thirds (which is the measure of his veracity) of 12. Thus, out of 15 instances, B is expected to be correct only in 8, when he reports what he has learnt from A. We see, then, that while the value of A's evidence is four-fifths, that of B's is eight-fifteenths or a little more than half. If, now, another individual C, whose veracity is $\frac{1}{2}$ reports what he has learnt from B (B having got his information from A), then the evidence is weakened to $\frac{8}{15} \times \frac{1}{2}$, *i.e.*, $\frac{4}{15}$. In such a case, there is greater reason to disbelieve C than to believe him, for the chance of his report being true is only 4 against 11. When testimony thus passes from generation to generation, as in the case of tradition,

its value evidently suffers to a great extent; and this explains, to a certain extent at least, our faith in the good old time or in the golden age.

Rule for determining the value of cumulative evidence.

§ 10. Rule for Determining the Cogency of Cumulative Testimony. *When several independent events conspire to support another, then the probability in its favour is measured by subtracting from unity the product of the improbabilities of these events.* It is evident, when several *independent* circumstances or witnesses support a fact, the probability in its favour is strengthened: and so the rule of multiplying their probabilities will not do in such a case, as the product would then be less than each separate fraction and would thus indicate a decrease, instead of an increase, of probability. With every additional evidence, the *improbability* diminishes; and hence the decrease of *improbability* is measured by multiplying together the fractions representing the separate *improbabilities.* We can thus determine the degree of *probability* in favour of the event by subtracting from unity (which represents absolute certainty) the product of the different *improbabilities.* If, for example, the reliability of two *independent* witnesses, A and B, be represented by $\frac{5}{6}$ and $\frac{4}{5}$, then their unreliability will be represented by $\frac{1}{6}$ and $\frac{1}{5}$ respectively. The combined unreliability of two such witnesses would, therefore, be $\frac{1}{6} \times \frac{1}{5}$ or $\frac{1}{30}$. The cumulative effect of their separate testimonies would accordingly be $1 - \frac{1}{30}$ or $\frac{29}{30}$, *i.e.*, 29 for and 1 against. It may be further explained thus: out of 30 cases, A will be right 25 times, (*viz.*, $\frac{5}{6}$ of 30),

Illustrations.

and in the remaining 5 times of 30, B will be right 4 times (*i.e.*, $\frac{4}{5}$ of 5). Thus, altogether both A and B will be right 29 times in 30. Similarly, if the probabilities in favour of three independent witnesses or circumstances be represented by $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{1}{2}$, then their separate improbabilities are represented by $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ respectively. The concurrence of these improbabilities is measured by their product, *viz.*, by $\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}$, *i.e.*, $\frac{1}{24}$. The probability, therefore, in favour of the fact supported by such *independent* witnesses or circumstances is $1-\frac{1}{24}$ or $\frac{23}{24}$. In other words, the odds in favour of it are 23 to 1. It may be mentioned in this connection that the value of analogical evidence as well as of circumstantial evidence is measured by this rule. In the case of analogical evidence, we are to determine the number of the points of community as well as the number of the points of difference and also the relative importance of each of these points (*Vide* Chap. XXII, §4); and then we are to estimate the strength of the analogical inference by subtracting from unity the product of the separate improbabilities. In the case of circumstantial evidence we should proceed in a like manner, giving each circumstance its due weight and then subtracting from unity the product of the different improbabilities. As circumstantial evidence occupies a very prominent place in the practical affairs of life, let us devote a separate section to its exposition and illustration.

The value of analogical and of circumstantial evidence is determined by this rule.

Evidence is a matter of fact tending to prove a

§ **11. Circumstantial Evidence.** Evidence may be said to be any matter of fact which goes to support a view or proposition. It is

generally taken to be of two forms—direct and indirect or circumstantial. The difference between the two forms of evidence is well indicated by Wills: "Circumstantial evidence is of a nature identically the same with direct evidence; the distinction is, that by Direct Evidence is intended evidence which applies directly to the fact which forms the subject of inquiry, the *factum probandum* [the fact to be proved]; Circumstantial Evidence is equally direct in its nature, but, as its name imports, it is direct evidence of a minor fact or facts, incidental to or usually connected with some other fact as its accident, and from which such other fact is therefore inferred. A witness deposes that he saw A inflict on B a wound, of which he instantly died; this is a case of direct evidence. B dies of poisoning; A is proved to have had malice and uttered threats against him and to have clandestinely purchased poison, wrapped in a particular paper, and of the same kind as that which has caused death; the paper is found in his secret drawer, and the poison gone. The *evidence* of these facts is *direct*; the facts themselves are *indirect* and *circumstantial*, as applicable to the inquiry whether a murder has been committed, and whether it was committed by A." (*An Essay on the Principles of Circumstantial Evidence*, pp. 16-17.)

position. It is either direct or indirect.

Character of circumstantial evidence.

As direct evidence generally rests on the testimony of the senses, it can scarcely be disputed, unless it is due to misapprehension or misrepresentation. Indirect or circumstantial evidence, however,

Direct evidence, resting on the testimony of the senses, cannot be disputed.

Indirect or circumstantial evidence is more or less uncertain, its probative force depending on the number and consistency of its several factors.

The rule given in the last section determines the value of circumstantial evidence, as circumstances are but 'inanimate witnesses.'

Circumstantial evidence is divided into three classes—violent, probable, and light—according to the degree of presumption created by circumstances. Illustrations.

is more or less uncertain; and its probative force depends mainly on the number and consistency of the several factors which go to support the thing to be proved. "The increase of force produced by the concurrence of independent *circumstances*," observes Wills, "is analogous to that which is the result of the concurrence of several independent *witnesses* in relating the same fact." (*Ibid.*, p. 279.) Hence facts and circumstances which tend to support a position have sometimes been called 'inanimate witnesses.' The rule, therefore, for determining the value of circumstantial evidence is the same as mentioned in the last section. It is not possible, however, in many cases to determine with numerical precision the probabilities of the several circumstances which go to support a view. Hence circumstantial evidence has broadly been divided into three main classes by reference to the presumptions created by them, *viz.*, into violent, probable, and light. "Upon an indictment for stealing in a dwelling house, if the defendant were apprehended a few yards from the outer door, with the stolen goods in his possession, it would be a violent presumption of his having stolen them; but if they were found in his lodgings some time after the larceny, and he refused to account for his possession of them, this, together with proof that they were actually stolen, would amount, not to a violent, but to a probable presumption merely; but, if the property were not found recently after the loss, as, for instance, not until sixteen months after, it would be but a light or rash presumption,

and entitled to no weight." (Archb. Crim. Plead., 208, 15th Ed.)

Circumstantial evidence, being generally uncertain, should always be examined with care. It may be accepted when nothing but the fact, which it is called upon to explain, can render a satisfactory account of it.

It should be remembered in this connection that circumstantial evidence being generally uncertain, we should consider it as satisfactory only when it cannot possibly be explained otherwise than by the fact which it is required to explain. "The force and effect of circumstantial evidence," says Wills, "depend upon its incompatibility with, and incapability of, explanation or solution upon any other supposition than that of the truth of the fact which it is adduced to prove; the mode of argument resembling the method of demonstration by the *reductio ad absurdum*." (*Op. cit.*, p. 18.)

A case illustrating how alternative hypotheses, explaining the circumstances adduced in evidence, may weaken their force.

The following case illustrates how even an instance which seems to justify violent presumption may be weakened by alternative hypotheses: "As an instance of violent presumption, amounting to convincing proof, Sir Edward Coke, and in this he is followed by several other authors of eminence, puts the case of one being run through the body with a sword in a house, whereof he instantly dies, and another man is seen to come out of that house with a bloody sword, and no other man was at that time in the house. "This", observes Chief Baron Gilbert, "is a violent presumption that he is the murderer; for the blood the weapon, and the hasty flight are all the necessary concomitants to such horrid facts; and the next proof to the sight of the fact itself, is the proof of those circumstances that do necessarily attend such fact." Notwithstanding the weight

of authority in its favour, this illustration of violent presumption has been made the subject of much and deserved observation. If the authors just quoted mean to say, as their words imply, that there is no possible mode of reconciling the above facts with the innocence of the man seen coming out of the house, the proposition is monstrous! Any of the following hypotheses will reconcile them, and probably others might be suggested. First, the deceased, with the intention of committing suicide, might have plunged the sword into his own body; the accused not being in time to prevent him, drew out the sword, and so ran out, through confusion of mind, for surgical assistance. Second, the deceased and the accused might have both worn swords; the deceased, in a fit of passion, attacked the accused; the accused, being close to the wall, had no retreat, and had just time enough to draw his sword, in the hope of keeping off the deceased, who, not seeing the sword in time, ran upon it, and so was killed. Third, the deceased may in fact have been murdered, and the real murderer may have escaped, leaving a sword sticking in or lying near the body, and the accused coming in might have seized the sword and run out to give the alarm. Fourth, the sword may have been originally a weapon of attack on the accused by the deceased, and wrenched from, and afterwards turned against him by the accused, under danger of attack on his life by pistol or otherwise." (Best, *Principles of the Law of Evidence*, 4th Ed., pp. 427-429.)

The above illustration brings out how circumstantial evidence, which at first sight may seem to be strong, may be weakened on close examination and cool reflection. The following considerations are mentioned by Wills as generally strengthening the force of this form of evidence:—"If it be proved that a party charged with crime has been placed in circumstances which commonly operate as inducements to commit the act in question,—that he has so far yielded to the operation of those inducements as to have manifested the disposition to commit the particular crime,—that he has possessed the requisite means and opportunities of effecting the object of his wishes—that recently after the commission of the act he has become possessed of the fruits or other consequential advantages of the crime,—if he be identified with the *corpus delicti* by any conclusive mechanical circumstances, as by the impressions of his footsteps, or the discovery of any article of his apparel or property at or near the scene of the crime,—if there be relevant appearances of suspicion connected with his conduct, person, or dress, and such as he might reasonably be presumed to be able, if innocent, to account for, but which, nevertheless, he cannot or will not explain,—if, being put upon his defence *recently* after the crime, under strong circumstances of adverse presumption, he cannot show where he was at the time of its commission,—if he attempt to evade the force of those circumstances of presumption by false or incredible pretences, or by endeavours to evade or pervert the course of justice,—the concurrence of

Circumstances determining the force of circumstantial evidence.

all or of many of these cogent circumstances, inconsistent with the supposition of his innocence and unopposed by facts leading to a counter presumption, naturally, reasonably, and satisfactorily establishes the moral certainty of his guilt,—if not with the same kind of assurance as if he had been seen to commit the deed, at least with all the assurance which the nature of the case and the vast majority of human actions admit." (*Op. cit.*, pp. 276-277.)

Laplace's theorem for the application of the Doctrine of Chances to the inductive determination of causes.

§ 12. The Application of the Theory of Probabilities to the Inductive Determination of Causes. It is thus explained and illustrated by Mill who borrows it from Laplace: 'Given an effect to be accounted for, and there being several causes that might have produced it, but of whose presence in the particular case nothing is known; the probability that the effect is produced by any one of these causes *is as the antecedent probability of the cause, multiplied by the probability that the cause, if it existed, would have produced the given effect.*' If, for example, an effect E might be produced by either X or Y, then the likelihood of its being produced by either of them is determined by the product of their relative frequency and relative efficiency. Three cases may be supposed to establish this position:—

Three cases.

(1) Let us imagine first that both X and Y are equally frequent, while X is doubly potent or efficient. Then, on a particular occasion, the chance of E being produced by X is double the chance of its being produced by Y. And

this rule, however uncertain with regard to an individual instance, is generally true in a large number of cases. To take a concrete case. Suppose that on any occasion deaths are produced by two diseases, plague and small-pox, and that they are equally frequent, though plague is twice as fatal as small-pox. Then, on an average, the number of deaths produced by plague would be twice as large as that produced by small-pox. And, with regard to any particular case, we may say that the chance of its being due to plague, as distinguished from small-pox, is as 2 to 1. (*Vide* § 3.)

(2) If, on the other hand, we imagine that both X and Y are equally potent, but Y is doubly frequent, then on any occasion, the chance of E being produced by Y is double the chance of its being produced by X. This rule is true also in average cases. If, in the concrete example given above, we suppose both plague and small-pox to be equally fatal, but the small-pox cases to be twice as many as those of plague, then, on an average, the number of deaths from small-pox would be twice as large as that from plague; and, with regard to any particular case of death, the chance of its being due to small-pox would be represented by the proportion 2 : 1.

(3) If we suppose X to be doubly frequent as well as efficient, then the chance of its producing E on any occasion will be measured by the product of the two proportions, namely, 2 : 1 and 2 : 1. In other words, the chance of E being produced by

X will be measured by 4 : 1. This is the measure of average cases. If, in the above concrete example, we suppose plague to be doubly fatal while small-pox thrice as frequent, then, on an average, the proportion of deaths from these two causes would be measured by the product of the two proportions 2 : 1 and 1 : 3, *i. e.*, by 2 : 3; and in any particular case of death, the chance of its being due to plague as against small-pox would be as 2 : 3.

By this theorem we may distinguish between the operation of law and that of chance in any case.

The above principle may be applied to distinguish *casual* coincidences from *causal* connections or those that result from law. The given fact may have originated either in an accidental conjunction of agencies or in a law of nature. The probabilities, therefore, that the fact originated in these two modes are determined by their relative frequency multiplied by their relative efficiency. But the peculiar combination of accidental circumstances, if it occurred, would be as much potent as a real law of nature. The probabilities, therefore, are determined by their relative frequency, which may be estimated by the Inductive Methods in the case of a law of nature, and by the calculation of Probabilities in the case of an accidental combination of agencies. (*Vide* Mill's *System of Logic*, BK. III, Chap. XVIII, § 5 and § 6.)

Safeguards against an improper use of Probability.

§ 13. Cautions against an Improper Use of Probability. Mr. Read lays down four principal cautions with regard to the use of probabilities:—"(1) Not to make a pedantic parade of numerical probability, where the numbers have

not been ascertained; (2) not to trust to our feeling of what is likely, if statistics can be obtained; (3) not to apply an average probability to special classes or individuals without inquiring whether they correspond to the average type; and (4) not to trust to the empirical probability of events, if their causes can be discovered and made the basis of reasoning which the empirical probability may be used to verify." (*Logic*, pp. 298-299.)

Probability is often misapplied to distort evidence.

It may be remarked in this connection that probability or approximate generalization is often an instrument of false reasoning in the hands of interested persons, such as advocates, orators, *etc.* By minimizing or excluding one set of circumstances, prominence is given to the other: negative instances may be suppressed and positive ones brought into relief, or *vice versa*. An advocate, for example, in defending his client, may conceal or minimize the circumstances leading to conviction; and thus, the circumstances favouring acquittal are made prominent. His client may thus be proved to be innocent, when really he may be the reverse.

§ 14. **Exercises.**

1. State precisely what is meant by Probability and by Probable Reasoning. How can the degree of the Probability of propositions be expressed?

2. Distinguish between Chance and Probability, and determine the grounds of each.

3. Explain and discuss the doctrine that Induction is based upon the Theory of Probability.

4. Explain the Theory of Probability and discuss its relation to Induction.

5. What do you understand by 'the average', and 'the personal equation'? Indicate their importance in the Theory of Probability.

6. What are the logical grounds for an estimate of Probability? Explain and illustrate the rule for estimating the value of traditional evidence.

7. What do you understand by Evidence? What are its different forms? Are they equally reliable?

8. What is meant by Circumstantial Evidence? State and explain the rule for estimating its value. Mention the circumstances which go to strengthen it.

9. Explain and illustrate the application of the Theory of Probabilities to the inductive determination of causes.

10. "Approximate generalizations give an opening to the bias of the feelings, and to the arts of a sophistical reasoner." How?

11. Mention the cautions to be observed to avoid an improper use of Probability.

12. What is meant by Chance? Give examples. How is it eliminated?

13. Give, with examples, the rules for the calculation of probabilities.

14. What is Probable Reasoning? Discuss the relation of Probability to Induction. State and illustrate the rules for the computation of probabilities.

15. Point out the importance of Statistics and their bearing on the Theory of Probability.

CHAPTER XXII. omitted 1 and 2 See

PROCESSES ALLIED TO SCIENTIFIC INDUCTION.

§ 1. Induction by Simple Enumeration. → omitted

Induction by Simple Enumeration rests on mere uncontradicted experience.

We have read that similarity underlies every form of inference ; and this is pre-eminently illustrated in the case of generalizations, whether hasty or careful. The value of a true inductive generalization rests on the careful examination of materials by the Inductive tests, without which a generalization becomes more or less precarious. In the case of Induction by Simple Enumeration an attempt is made to establish a general proposition on the strength of uncontradicted experience. This, as we have seen, is the popular form of generalization. Common people never take the trouble of carefully examining cases before arriving at general propositions ; they merely observe certain cases, having some peculiarity or feature, and hastily conclude therefrom that all like cases are characterised by the same peculiarity or feature. This tendency to hasty generalization is specially strong in the untutored mind. Persons trained to scientific inquiry or logical examination take the trouble of carefully sifting the materials by the logical methods before establishing a general truth. (*Vide* Chap. XVI, § 7 and Chap. XVIII, § 2.)

Induction by Simple Enumeration is quite in

harmony with the general lethargy and passive attitude of the mind. The employment of the Inductive Canons is more or less laborious and implies an intelligent manipulation of materials. The great merit of Bacon is to have pointed out the superiority of active manipulation of materials to their mere passive contemplation. According to him, the active interrogation of Nature by Experiment is always an essential condition of inductive generalization.

It is usually precarious;

It is apparent from the preceding remarks that Induction by Simple Enumeration is not so conclusive as Scientific Induction. In the former, the causal connection is not proved by laborious research; it is merely assumed. When, therefore, a generalization is reached without adequate examination, it is of an uncertain character. But certainty of this form of Induction increases as the chances of exception decrease. In fact, the ultimate laws of Nature are proved finally by Induction by Simple Enumeration, which underlies what Bain calls Universal Agreement through all Nature. According to empiricists, the truth even of the Law of Causation or of the Uniformity of Nature is proved in this way. Such a law, they say, is believed to be certain because no exception has hitherto been noticed in its operation and the chances of exception are few. Had there been any exception, it is urged, it must have been known to some individual or individuals at some time or place; but the very fact that no exception has hitherto been found creates a presumption in favour

but its strength increases with the multiplication of experience.

of the universal truth of the Law. (*Vide* Chap. II, § 2 and Chap. XVII, § 3.) The value of this form of Induction depends, as Fowler points out, on "(1) the number of positive instances which have occurred to us; (2) the likelihood, if there be any negative instances, of our having met with them." And, as he observes, "The first of these considerations deserves little weight, unless supported by the other." (*Inductive Logic*, p. 207.)

Its value is determined by the positive and negative instances and by the degree and extent of knowledge in any sphere.

omitted

Geometrical reasoning, as we have seen, is deductive; but the law of a series in mathematics, when proved in innumerable cases, may be taken to illustrate Induction by Simple Enumeration.

§ 2. **Mathematical Induction.** We considered in Chapter XV, § 5, certain forms of apparent Induction which are really deductive in character. We have seen that the demonstrations of Euclid and the questions of identity to establish a minor have sometimes been regarded as Inductive. But these forms of reasoning are essentially deductive, since they follow from general principles, previously known. Nevertheless, there are some forms of mathematical reasoning which may be regarded as Inductive.

When, for example, we observe that a series of odd numbers, added together from the beginning, gives a sum which is equal to the square of the number of odd numbers in the series, then the result is first established by reference to individual instances. Numerous instances of such observation lead us to generalize the law, that the sum of the odd numbers is equal to the square of the number of terms in the series.

Thus, $1+3=2^2$,

$1+3+5=3^2$,

$1+3+5+7=4^2$,

$$1+3+5+7+9=5^2,$$
$$1+3+5+7+9+11=6^2.$$

If we take n number of odd numbers, the sum would be—

$$1+3+5+7+\cdots+(2n-1)=n^2.$$

By adding $2n+1$ (which represents the next odd number) to each side of the equation, we get

$$1+3+5+7+\cdots+(2n-1)+(2n+1)=n^2+2n+1$$
$$=(n+1)^2.$$

Thus, the law being true for n terms, it is proved to be true also* for $n+1$ terms; and so on we can proceed. Hence, the law may be extended to cover all similar cases.

This argument cannot be called strictly inductive, in as much as we do not employ the inductive canons to arrive at the result. But though such a form of Induction falls short of Scientific Induction, yet it may be regarded as a form of Induction by Simple Enumeration. The inference evidently is not so certain as what we find in Scientific Induction, where the law of causation is taken as the formal ground. But, still, the inference is to a great extent certain, in as much as no exception to the law has hitherto been noticed; and had there been any, it would have been known to some persons somewhere. Thus, such mathematical inductions approach to a great extent the certainty of the ultimate laws of nature proved by Universal Agreement. The rule of the Binomial

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* The first odd number is $2-1$; the second, $2\times2-1$; the third, $2\times3-1$; the 4th, $2\times4-1$; and so the nth is $2n-1$; and the $(n+1)$th is $2(n+1)-1$, *i. e.*, $2n+1$.

Theorem may also be similarly viewed. Geometrical truths reached by an examination of many individual instances illustrate likewise this form of Induction. (*Vide* Chap. XV, § 5 and *Elements of Psychology*, Chap. XIII, § 10.)

§ **3. Analogy.** Analogy may be defined as a kind of probable reasoning in which we infer that things resembling each other in certain respects resemble also in other respects, though no causal connection is known to exist between the points of resemblance and the inferred quality or qualities. In it we rely upon some such vague notion of uniformity as that "things alike in some respects are also alike in others." When, for example, we observe that the planet Mars has a similar atmosphere to that of the earth and that there is a like distribution of land and water, of heat and cold, *etc.*, we conclude by Analogy that Mars also may be inhabited by beings like what we find on earth. Every argument rests on similarity: but in Analogy there is an imperfect or inadequate similarity among the data to justify a result; perfect similarity, as we have seen, implies identity of essence, justifying conclusive proof. (*Vide* Chap. II, § 4.) Analogy is called by Gotama *Upamanam* (उपमानम्) from *Upama* (उपमा) resemblance.

Analogy defined.

Analogy is based on imperfect similarity.

Points of difference between Induction and Analogy:

(1) The principal difference between Induction and Analogy lies in the fact that in the former a (causal) connection is known to exist between the inferred property or feature and the ground of inference, while in the latter no such connection is

(1) Induction is based on causal connection, while Analogy is not so.

known to exist. If, for example, we observe two objects X and Y and, finding that they resemble in *a* and *b*, we conclude that they resemble also in *c*, which is found in X, the argument is inductive or analogical, according as a causal connection is known or not known to exist between *a* and *b* on one side and *c* on the other. The one is based on the uniformity of causation, while the other on that of mere co-existence. And this explains why induction is comparatively certain, while analogy is only more or less probable. Since in analogical argument no connection is known to exist between the inferred property and the data, we can never state as the principle of reasoning a general proposition, which we virtually do in induction.

(2) In Induction we pass from cases to a law, while in Analogy from instance to instance.

(2) In Induction we proceed from individual instances to a general law, but in Analogy we proceed from particular to particular, without the help of a general law.*

* Analogy is often used when there are only two things, the one furnishing the ground of inference and the other, its subject. We may thus infer that Venus or Mars is inhabited by comparing it with our own globe; or we may think that a definite line of inquiry which succeeded in one case will succeed in another, owing to its great resemblance with the other. In the case of Induction, on the other hand, we pass from several instances to a general law justified by all of them. In analogy the denotation is small but the connotation is large, for without many points of similarity we are not justified in proceeding from one case to another; while, in Induction, the denotation is wide, but the connotation narrow, since it ordinarily aims at establishing a relation between two qualities or features. "Induction", says Bowen, "proceeds upon the principle, that *what certainly belongs to many Individuals of the same kind*, *also probably belongs to all the other individuals of that kind*; the principle of Analogy is, that, *if two things agree in many respects, they probably agree also in some other respect* Because *some one* quality *exists in many* things, therefore it *exists in all* of the same kind; that is Induction. Because *many* qualities in this are

(3) In Induction we employ the Experimental Methods to be sure of the causal connection, which we try to generalize. In Analogy, on the other hand, we proceed from one case to another merely on the ground of some points of similarity, without the application of any Inductive Test.

(3) In Induction we employ the Experimental Methods, while in Analogy we do not.

Analogy has sometimes been described as "similarity in relations." A wave of water, for example, has been likened to the undulation of air. This may be expressed thus, 'a wave is to water, as an undulation is to air.' Similarly, it may be said that a sovereign is to his subjects, what the head of a family is to its members. But this account of Analogy is at best a description; it does not explain the essential character of analogical argument. It is too vague to be of any practical value.*

The description of Analogy as 'similarity in relations' is not precise.

§ 4. Strength of Analogical Argument.

Analogical argument being based on imperfect similarity is necessarily of a probable character. As we are not aware of a causal connection be-

Analogical argument is only probable,

the same as in that, therefore *one* other quality in this resembles that; this is Analogy. In other words, induction concludes from *one in many* to *the others*, by way of Extension; Analogy, from *many in one* to *the others*, by way of Intension." (*Logic*, p. 381.)

* Analogy (Gr. *analogia*—*ana*, according to, and *logos*, ratio, proportion) was originally used by Aristotle to express equality of ratios. It thus corresponds to what is known as Proportion in arithmetic. Thus 3 : 6 : : 12 : 24 ; or, as health : the body : : virtue : the soul. Hence, in ordinary discourse, we speak of analogy when we have before us two pairs of things and there is resemblance between their relations. Whately evidently had this meaning in view when he defined Analogy as "resemblance of *ratios* or relations." (*Logic*, p. 123.) The modern sense of the term, though comparatively loose, is connected with its original sense: whenever we draw an inference by analogy, we do so owing to the identity or similarity of relation between the known and unknown properties in the two cases.

tween the inferred quality and the data, we can never be certain that the latter being present, the former must be so also. Though analogical argument is probable in character, yet the degree of probability is not always the same. *The force of analogical argument depends on the number and importance of the points of similarity, as distinguished from the points of difference.* As Dr. Bosanquet puts it, in analogy we must *weigh* the points of resemblance, not simply *count* them. In estimating the strength of analogical argument, we must also take into our consideration the number of properties unknown to us; for *the relative proportion of the known to the unknown properties would affect the force of Analogy.* If the presumption is that numerous points are unknown, the argument must be weak; if, on the other hand, a relatively large number of points is known, the argument is comparatively strong, provided the points of community are important or essential. This is sometimes expressed mathematically by the rule that the value of an analogy may be represented by a fraction having as its numerator the resemblances between the two things compared and as its denominator the differences between them *plus* the number of qualities of which we are ignorant regarding them. The rule, however, should not be taken to indicate an exact mathematical ratio, which is often hard to determine in view of the difficulty of balancing the importance of qualities with their number. It suggests only the general relation that the relative proportion of

the degree of probability being determined by the number and importance of the points of similarity

as well as by the proportion of known to unknown qualities.

the points of similarity and known points to the points of difference and unknown points determines the force of analogy in any case.

Illustrations.

I have heard of a physician who advised his patients never to have a bath. His theory was that if an object be alternately dry and wet, it wears out sooner than if it be always dry. And he justified his position by analogy. Cut, for example, a rope into two halves and use one portion for drawing water and the other for hanging clothes. Of these two parts, the latter would evidently last longer, it being always dry. It was argued, the human body, when always dry, must continue longer unimpaired than when it is alternately dry and wet, as in the case of the daily bath. The fallacy lies here in the false analogy between a cord and the human organism: the two resemble only in being material; but while the one is inanimate, the other is endowed with life. The points of difference here are too important and numerous to justify an analogical inference. The value of the following argument, used in *The New Science of Healing Without Medicine*, in favour of a cold bath after a hot one can easily be determined by the reader:—"Steel, when brought to white heat in the fire, must be plunged into cold water in order to obtain the requisite temper. Similarly the human body after the steam-bath, on being cooled down, becomes strong and hardy." (Eng. trans. of 1905, p. 104.)

It is argued likewise that, since individuals pass through the three stages of growth, vigour,

and decay, a community must also do the same. The argument is based on a false analogy between the life of a man and the progress of a community. Sir G. C. Lewis well observes on this point :—"The comparison which is sometimes instituted between the progress of a community and the life of a man fails in essentials, and is therefore misleading. Both a man and a community, indeed, advance from small beginnings to a state of maturity ; but a man has an allotted term of life, and a culminating point from which he descends ; whereas a community has no limited course to run ; it has no necessary period of decline and decay, similar to the old age of a man ; its national existence does not necessarily cease within a certain time. Nations, as compared with other nations, have periods of prosperity and power ; but even these periods often ebb and flow, and when a civilised nation loses its pre-eminence—as Italy in the nineteenth, as compared with Italy in the fourteenth and sixteenth centuries—it does not necessarily lose its civilisation. A political community is renewed by the perpetual succession of its members ; new births, immigrations, and new adoptions of citizens, keep the political body in a state of continuous youth. No such process as this takes place in an individual man. If he loses a limb, it is not replaced by a fresh growth. The effects of disease are but partially repaired ; all the bodily and mental functions are gradually enfeebled, as life is prolonged, till at last decay inevitably ends in death ; whereas a community might, consistently with the laws

of human nature, have a duration co-extensive with that of mankind.

"The supposed analogy between the existence of a political community and the life of a man seems to have contributed to the formation of the belief in a liability to *corruption*, inherent in every society. It was a favourite doctrine among some writers of the last century, that every civilised community is fated to reach a period of corruption, when its healthy and natural action ceases, and it undergoes some great deterioration. The notion of an inevitable stage of corruption in a nation was, indeed, partly suggested by the commonplaces condemnatory of luxury, derived both from the classical and ecclesiastical writers; and by the more modern eulogies of a savage life. So far, however, as it was founded on the inevitable periods of decay in animal and vegetable life, the comparison was delusive; for the two relations which are brought together do not correspond. The death of individuals may, indeed, be considered a necessary condition for the progress of the society, into which they enter as temporary elements. It is by the substitution of new intelligences, and of natures not hardened by old customs, for minds whose thoughts and habits have learnt to move uniformly in the same groove, that progressive changes in human affairs are effected. The decay and death of the individual, therefore, tends not only to prevent the deterioration of the society, but to promote its improvement." (*Methods of Observation and Reasoning in Politics*, II, p. 438.)

Analogy is a fruitful source of hypotheses.

§ 5. Analogy as a Source of Discovery and Means of Proof. We have seen that Analogy is a fruitful source of hypotheses and so of Discovery. (*Vide* Chap. XIX, § 2.) Striking resemblances between things often lead men to imagine that one law weaves them together and thus to start a hypothesis calculated to explain them and all similar cases. And it is here that we find the difference between genius and common intelligence. While men of true imaginative insight and sound judgment patiently arrive at valid hypotheses by a careful and comprehensive estimate of facts, men of weak imagination and of shallow judgment hastily frame extravagant hypotheses only by a narrow and superficial view of things. And at times we find analogy in a ludicrous form illustrated in the far-fetched metaphors or similes of wit, as when one mentions that the strength of an elephant may be found in a mosquito, since both are animals having legs and a proboscis. The importance of Analogy, as an instrument of discovery, depends, therefore, on its cautious use by reference to the characteristics of the facts which are compared together. The truth of these remarks will appear from the following illustrations :—

Illustrations.

(1) Bishop Wilkins quotes the following from Cardinal Nicolo de Cusa :—"We may conjecture the inhabitants of the sun are like to the nature of that planet, more clear and bright, more intellectual than those in the moon where they are nearer to the nature of that duller planet, and those of

the earth being more gross and material than either, so that these intellectual natures in the sun are more form than matter, those in the earth more matter than form, and those in the moon betwixt both. This we may guess from the fiery influence of the sun, the watery and aerous influence of the moon, so also the material heaviness of the earth. In some such manner likewise is it with the regions of the other stars; for we conjecture that none of them are without inhabitants, but that there are so many particular worlds and parts of this one universe, as there are stars, which are innumerable, unless it be to Him who created all things in number." (*Discovery of a New World in the Moon*, p. 128.)

(2) Dr. Reid writes:—"We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus, and Mercury. They all revolve round the sun, as the earth does, although at different distances and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve round their axis like the earth, and by that means, must have a like succession of day and night. Some of them have moons, that serve to give them light in the absence of the sun, as our moon does to us. They are all, in their motions, subject to the same law of gravitation, as the earth is. From all this similitude, it is not unreasonable to think, that those planets may, like our earth, be the habitation of various orders of living creatures. There is some

probability in this conclusion from analogy." (*Intellectual Powers*, Chap. IV, Hamilton's Ed., I, p. 236.)

(3) Dr. Bain observes :—"Much speculation has been expended on the question—Are the planets inhabited? The argument is at best analogical; and there is not even the force of analogy except with reference to a small number. Bodies, like the moon, possessing no water and no atmosphere, must be dismissed at once. The planets generally appear to possess atmospheres. We seem justified, however, in making a summary exclusion of the near and the remote planets on the ground of temperature. All organized life known to us, is possible only within narrow limits of temperature; no animal or plant can exist either in freezing water or in boiling water. Now, the temperature of Mercury must in all likelihood be above the boiling point, even at the poles, and the temperature of Uranus, and of Saturn, below freezing at the equator. The constituent elements being now shown to be the same throughout the solar system—Carbon, Oxygen, Hydrogen, etc., we are not to presume any such departure from our own type of organized life as would be implied by animals and plants subsisting in these extremes of temperature. On the supposition that the sun's temperature has steadily decreased, and is still decreasing, by radiation, the day of living beings is past for Uranus and Saturn, and perhaps for Jupiter; it is not begun for Mercury.

"Confining ourselves, therefore, to the neigh-

bouring planets, and referring to the others only for the periods, past or future, when the capital circumstance of temperature is suitable, we have an analogical argument as follows. Venus and Mars are gravitating masses like the earth, containing, we may now say with certainty, the same materials as this globe—solid, liquid, and gaseous. But we cannot tell the precise arrangement of the constituent substances; and, seeing that with ourselves so much depends upon the mere collocation and amount of such elements as oxygen and carbon, we may consider that the unknown properties of the supposed planets are considerable in number, and serious in character. The probability arising out of the points of agreement, if not greatly affected by known differences, is reduced by this large element of the unknown." (*Logic*, II, p. 147.)

The above illustrations show how prone we are to frame hypotheses by analogy, though the ground for such a procedure may not be strong in all cases. In the first example (1), given above, there is only one point of similarity, while there are several and important points of difference; and dissimilarity is made here the ground of inference. It is arbitrarily assumed that the composition of the sun, the moon, and the earth is unlike and that the degree of intelligence is connected with the 'clearness and brightness' of the sphere in which it is found. In (2), several points of similarity are noted, but the points of difference and the unknown qualities are passed over. In (3),

the points of difference and unknown possibilities are emphasized to such an extent as to exclude all reasonable hypotheses. These instances bring out that the value of analogical inference, as a mode of proof, is generally very low. And this is evident from the fact that the inference is ordinarily based on superficial points of similarity, since we are ignorant of a causal connection. It never approaches certainty and often gives rise to very weak probability. "The degree of probability," says Minto, "is much nearer zero than certainty." (*Logic*, p. 369.) And the following analogical argument, employed by the advocates of annual Parliaments in the time of the Commonwealth with reference to the serpent's habit of annually casting its skin, is aptly quoted by him:—

As a mode of proof, its force is very weak.

"Wisest of beasts the serpent see,
Just emblem of eternity,
And of a State's duration;
Each year an annual skin he takes,
And with fresh life and vigour wakes
At every renovation.
Britain! that serpent imitate,
Thy Commons House, that skin of State,
By annual choice restore;
So choosing thou shalt live secure,
And freedom to thy sons inure,
Till Time shall be no more."

Analogy is thus concerned rather with discovery than with proof: it enables us readily to frame hypotheses, but does not supply definite

tests by which we can verify them. It suggests a line of inquiry, but cannot finally settle it. "In all cases," says Lotze, "when we believe we can prove by analogy, the analogy in fact is distinctly not the ground of the conclusiveness of the proof; it is only the inventive play of thought by which we arrive at the discovery of a sufficient ground of proof." (*Logic*, § 214.) What are known as happy hits or lucky guesses of natural sagacity or trained intelligence are often due to such analogical inference. Thus, in the practice of medicine or the art of mining happy results have sometimes been achieved by striking similarities. It is said that a general resemblance of the hills near Ballarat in Australia to the Californian hills where gold had been found led to the discovery of gold at Ballarat. When in an analogical reasoning (1) the resemblances are very great, (2) the points of difference very small, and (3) our knowledge of the subject-matter is tolerably wide, then the force of such an inference approaches very near to Induction. And Induction, Explanation, and Analogy are thus very closely related. Newton's discovery of universal gravitation from his assimilating the fall of a stone to the deflection of the moon towards the earth or his inference that the diamond is combustible from his knowledge that combustible bodies (such as camphor, amber, olive oil, linseed oil, spirit of turpentine) have unusual refractive power and that diamond also is a highly refracting body, is sometimes attributed to analogy. But these are rather instances of extended generalization. Had

It often leads to discoveries.

Analogy, Induction, and Explanation are closely related.

the inference been from a single body, as a stone or an oil to the moon or the diamond, the argument might be construed as analogical. But the inferences were arrived at after comparing several bodies (such as the attraction of the planets to the sun and of the moon to the earth in the one case and the highly refracting power of many unctuous and sulphureous bodies in the other); and so these are rather instances of wider generalization. "The suggestion as to the diamond", as Bain observes, "arose from its position among a number of highly refracting bodies that agreed in being of an inflammable or combustible nature. The concurrence of high refracting power with inflammability was an empirical law; and Newton perceiving the law extended it to the adjacent case of the diamond. The remark is made by Brewster that had Newton known the refractive powers of the minerals *greenockite* and *octohedrite*, he would have extended the inference to them, and would have been mistaken." (*Induction*, pp. 144-145.)

A case is presented which is made the basis of generalization.

§ 6. **Value of Examples.** We may discuss in this connection the value of Examples or Instances, which we often cite in the course of an inquiry or exposition. When, for example, a chemist finds by analysing a sample of water that it contains eight parts of oxygen and one part of hydrogen, and he concludes therefrom that water everywhere and always is so composed, he evidently takes a step from the known to the unknown, which is the most important mark of Induction. Similarly, in

explaining a particular subject, say Botany, a particular leaf or plant may be examined and its properties discovered. When, on the strength of such observation, generalization is made with regard to all such leaves or plants, the march of reasoning is from a single or a few cases to all like cases. The argument, therefore, in all such cases, is apparently inductive. Whether, however, such an argument is to be regarded as strictly inductive, from the scientific stand-point, depends on the character of the relation existing between the known and the unknown properties. If there is a causal connection between the inferred property and the known features, and the result is arrived at by the application of the inductive tests, then the argument may be regarded as strictly inductive. If, on the other hand, no causal connection is known, then the argument may amount only to analogy or, at best, to induction by simple enumeration, when instances are multiplied.

Such a procedure illustrates either induction or analogy.

§ 7. **Exercises.**

1. Determine the character of Inference by Simple Enumeration and indicate the circumstances on which its value depends. How is it related to Scientific Induction ?

2. What do you understand by Mathematical Induction ? Is it strictly inductive ? Estimate its cogency.

3. Explain the nature of the argument from Analogy, stating the conditions on which its force depends.

4. What has argument from Analogy in common with, and wherein does it differ from, Deduction and Induction ?

5. Analogy has sometimes been defined as 'resemblance in relations.' Is the definition correct ?

6. Show, with illustrations, the place of Aanalogical Reasoning in the process of scientific discovery.

7. How would you distinguish a sound from an unsound Analogy? Give illustrations. Can an analogical argument be ever regarded as conclusive?

8. Explain the use of Examples in inductive reasoning and determine their force as instruments of proof.

9. Why is a single instance sometimes sufficient to warrant a universal conclusion, while in other cases the greatest possible number of concurring instances, without any exception, is not sufficient to warrant such a conclusion?

10. Examine the value of the following arguments:—

(*a*) England has a democratic franchise, therefore India should have a democratic franchise too.

(*b*) All the great empires that have ever existed have lost their position of eminence, hence no great empire in the future will maintain its supremacy.

(*c*) A sovereign : the state : : a pilot : the ship.

(*d*) A nation must ultimately perish because it is an organism, and all organisms grow old and die.

(*e*) The metropolis of a country is similar in many respects to the heart of the animal body, therefore the increased size of the metropolis is a disease.

(*f*) Nobody can be healthy without *exercise*, neither Natural Body, nor Body Politic: and certainly, to a Kingdom or State, a just and honourable war is the true *exercise*. A civil war, indeed, is like the heat of a fever, but a foreign war is like the heat of *exercise*, and serves to keep the Body in health.

(*g*) A nation, like an individual, must pass through periods of growth, maturity, and decay.

(*h*) Is not dirt washed away by a current of water? Yes. Then, is it impossible that all the sins of omission and commission may be washed away by the holy water of the Ganges when one dips into it? No. Thus, it matters little how one acts or thinks so long as he periodically bathes in the Ganges.

11. 'A house without tenant, a city without inhabitants, present to our minds the same idea as a planet without life,

a universe without inhabitants.' The conclusion here evidently is that the planets and stars are inhabited. What is the logical form of the inference ? State it in its simplest form. What do you consider to be its logical value, and why ?

12. Estimate the force of the following argument :—

"When a tree, or a bundle of wheat or barley straw, is burnt, a certain amount of mineral matter remains in the ashes—extremely small in comparison with the bulk of the tree or of the straw, but absolutely essential to its growth. In a soil lacking, or exhausted of, the necessary mineral constituents, the tree cannot live, the crop cannot grow. Now contagia are living things, which demand certain elements of life just as inexorably as trees, or wheat, or barley ; and it is not difficult to see that a crop of a given parasite may so far use up a constituent existing in small quantities in the body, but essential to the growth of the parasite, so as to render the body unfit for the production of a second crop. The soil is exhausted, and, until the lost constituent is restored, the body is protected from any further attack of the same disorder. Such an explanation of non-recurrent diseases naturally presents itself to a thorough believer in the germ theory......To exhaust a soil, however, a parasite less vigorous and destructive than the really virulent one may suffice ; and if, after having by means of a feebler organism exhausted the soil, without fatal result, the most highly virulent parasite be introduced into the system, it will prove powerless. This, in the language of the germ theory, is the whole system of vaccination." (Tyndall.)

Division III.

RESULTS OF INDUCTION.

CHAPTER XXIII.

LAWS OF NATURE.

§ 1. **Science and Law.** The relation of Science to Law is very close. In order to understand this relation, let us first try to comprehend what is meant by a 'Law' and a 'Law of Nature.'

Definition of 'Law.'

A Law is but the expression in language of some uniform relation existing among the phenomena of a particular class.

Illustrations.

For example, the Law of Gravitation enunciates a uniform relation existing among material bodies, and the Law of Definite Proportions similarly explains a uniform relation existing among chemical elements.

Laws are either human or natural.

A Law, however, may be either human or natural. In the case of a human law, we find the same aspect of uniformity; such a law is imposed by a sovereign upon his subjects for the uniform regulation of their conduct. Natural Laws, similarly, imply uniform relations existing among phenomena, but not established by human authority.

Natural Laws are of various kinds, governing different departments of Nature.

Natural Laws are necessarily of various kinds as governing different departments of Nature; there are thus Physical, Chemical, Mathematical, Logical, Mental, and Moral Laws. As Bain puts it, "The course of the world is not *a* uniformity, but *uni-*

formities. There are departments of uniformity, which are radically distinct." (*Logic*, II, p. 8.) That is, the course of Nature is made up of several uniformities expressed in several laws.

Without laws there would be no room for science.

From the above account of the character of laws it is patent that the connection between Law and Science is very intimate. If there be no law, if Nature be capricious in her conduct, then evidently there will be no room for knowledge or expectation, and consequently none for science. If everything be in a chaotic condition, without any order or system, then evidently there would be no fixed rule which science would try to discover. The different sciences are thus but expositions of the different kinds of uniformity prevailing in the different departments of Nature.

The laws ultimately rest on the Uniformity of Nature, which we instinctively recognise.

A question may be raised here with regard to the *ultimate ground of these laws.* Reflection shows that laws finally presuppose that Nature is uniform in her operation. The Uniformity of Nature is thus the ultimate postulate on which all Laws rest. But if it be further asked, what is the ground of this Uniformity of Nature itself, then the reply may entangle us in a circle. If, for example, we hold that the Uniformity of Nature is proved by the several laws (for Nature is uniform here, there, and everywhere), then we move in a never-ending circle. Hence, the objection of Mansel against the empirical origin of the Law of Causation applies with no less force to a similar explanation of the Law of Uniformity of Nature. (*Vide* Chap. XVII, § 10.) The Law seems to be the

expression of an instinctive tendency to generalize on the model of present experience; it is due to what Bain calls 'the mere instinct of generalization.' (*Induction*, p. 113.) [*Vide* Chap. II, § 7.]

The world is a systematic whole including several laws harmoniously adjusted to serve some supreme end.

§ 2. **The World as a System of Laws.** The world, as conceived by us, is a well-ordered system, the different parts of which are harmoniously related to one another. We, accordingly, find that the laws which are special to a particular subject are not altogether unconnected with the laws which hold good in the other departments of Nature. There is a close connection between, say, Physical and Chemical laws, Chemical and Biological laws, Biological and Psychological laws, Psychological and Sociological laws, and Sociological and Moral laws. The modern doctrine of conservation of energy has established beyond dispute that one form of energy may be transformed into another, indicating a correspondence among the different laws. We not only find that the laws of the different sciences are closely connected with one another, but we also find that, within one and the same department, the different laws are interconnected. The world is thus a unity viewed as a whole as well as in all its parts: it beams with intelligence and beauty in every detail, no less than in its entire mechanism. It is not a chaos, but a cosmos. There are laws within laws—some more general and some less—so that to the Omniscient Mind the cosmos is compressed in a nut-shell of a few wide or comprehensive laws. The world is thus a type of Beauty, Harmony, and Consistency.

§ 3. Classification of Laws. Laws are classified, according to the degree of generality, into higher and lower, though their grades may not be clearly distinguishable.

Laws are classified into three groups according to the degree of their generality : (1) Axioms, which are ultimate and self-evident principles, constituting the foundation of all inference.

(1) The most general laws, which are viewed as universal and self-evident, are known as *Axioms*. They rest on their own evidence and are thus viewed as the ultimate principles on which all arguments depend ; they are, moreover, considered to be the goal of all generalization. Such principles are the Laws of Identity, Contradiction, and Excluded Middle, the Axioms of Mathematics, and the Law of Causation. Logic assumes them, leaving it to Metaphysics to examine and explain their nature. (*Vide* Chap. III, § 4.)

Examples.

(2) Primary or Ultimate Laws, which are the highest generalizations from experience.

(2) The laws which are next in order of generality are called *Primary* or *Ultimate*. Their sphere also is extensive, though not so wide as that of the Axioms. They being thus of less wide scope may be proved by the axioms. Such laws are the Law of Relativity in Psychology, Definite Proportions in Chemistry, Gravitation in Astronomy, *etc.*

Examples.

(3) Secondary Laws, which are applicable to a special group of facts.

(3) The laws which are comparatively special in character are known as *Secondary* (called by Becon the 'Middle Axioms' or 'Intermediate Generalities'), they being but steps for rising to the supreme laws. They are in touch with concrete circumstances, and are thus of greater service in the practical affairs of life than the Axioms or Primary Laws. As Secondary Laws relate to complex situations, they involve combinations of

They are of great practical value.
They are due to the

convergence or conflict of Primary Laws.

several Primary Laws governing the constituent factors or elements.

Secondary Laws have been classified into different groups : (*a*) Laws are Derivative or Empirical, according as they follow or do not follow from Primary Laws. Empirical Laws rest mainly on the Method of Agreement.

Secondary Laws have been classified into different groups according to different principles of classification :—

(*a*) Secondary Laws have been divided into *Derivative* and *Empirical*,* according as they are deduced from higher (Primary) laws, or as they rest on mere experience, *i.e.*, on a detailed examination of facts. Empirical laws rest mainly on the Method of Agreement. It is a question whether laws proved by Difference should be considered as Empirical or Derivative. In a certain sense, no doubt, laws thus proved may be considered as Derivative, in as much as they are based on the law of Causation. But, it seems desirable that such laws should be further proved by Primary or Ultimate laws before they can be so considered.

(*b*) Laws are invariable or approximate, according as they govern an entire class or only a part of it. Approximate generalizations also are useful in the practical affairs of life.

(*b*) Secondary Laws have also been divided into *Invariable* and *Approximate Generalizations*, according as they express general relations without any exception (within the limits of our experience) or as they stand for partial truths valid in most cases. Approximate generalizations, though short of universal truth, are also useful in the practical affairs of life. Their utility is heightened when they can be reduced to a definite form by reference

* The word 'Law' implies necessary connection, while 'empirical', implying what is begotten of experience, indicates mere association. Thus, what the substantive affirms, the qualifying adjective practically denies. The expression 'Empirical Law,' accordingly, seems to be a misnomer or contradiction in terms. Mere empirical generalizations can never be said to possess the necessity (natural or artificial) which is to be found in 'laws'. As, however, the expression is sanctioned by usage, it is adopted here.

to percentage or proportion. In politics, for example, such approximate generalizations are of great value: all that a legislator can reasonably aim at, must be an approximate result, instead of a strictly general one. And we have also read that Probability ultimately rests on Approximate Generalizations. (*Vide* Chapter XXI, § 5.)

(*c*) Secondary laws may further be sub-divided into those of either (*A*) *Succession* or (*B*) *Co-existence.*

(*c*) Laws express a relation of either (*A*) Succession or (*B*) Co-existence.

(*A*) Secondary laws expressing *Succession* may refer to either

(*A*) Forms of Laws of Succession.

(1) Causation (*e.g.*, fire consumes fuel), or

(2) the effect of a remote cause (*e.g.*, good rain brings a good harvest), or

(3) the joint effects of the same cause (*e.g.*, the succession of day and night).

(*B*) Secondary Laws expressing *Co-existence* may refer to—

(*B*) Forms of Laws of Co-existence.

(1) Comparatively general laws based on agreement (*e.g.*, gravitating bodies are inert);

(2) Co-existence of properties in Natural Kinds (*e.g.*, the numerous properties which co-exist in gold);

(3) Co-existence of qualities not essential to a species (*e.g.*, flowers of scarlet colour have no smell);

(4) Constancy of relative position (*e.g.*, the position of planets in the solar system, the sides and angles of a rectilineal figure).

Most of the relations of Co-existence are, on careful examination, reducible to Laws of Causation. When, however, such relations cannot be

Many relations of co-existence are reducible to causation.

derived from causation, they can be proved only by collecting numerous examples and relying mainly on the Uniformity of Nature. (*Vide* Chap. XVI, § 2.)

A subordinate law when deduced from higher laws is called a Derivative Law.

§ 4. Derivative and Empirical Laws, and Forms of the Latter. As explained in the preceding section, when a subordinate law is deduced from higher laws, it is considered as Derivative. When, for example, the law of terrestrial gravitation is deduced from the law of universal gravitation, then the law of terrestrial gravitation is to be considered as Derivative. An Empirical Law, as already remarked, rests only on the evidence of experience. It is known, for example, that 'white tom-cats with blue eyes are deaf,' that 'the fall of the barometer indicates wind or rain.' And almost all laws are in the first instance of an empirical character before they are traced up to higher laws. Thus, that iron rusts, that explosion follows the contact of a spark with gun-powder, that a storm follows the appearance of a circle round the moon are empirical generalizations which may or may not turn out to be derivative according as we succeed or fail to discover higher laws to which they may be traced. Empirical Laws may be of different forms, three of which deserve notice :—

A Empirical law rests only on experience.

A law is usually first empirical before it becomes derivative.

***Forms of Empirical Laws*:**

(1) An empirical law believed to be deducible from higher laws.

(1) An empirical law applicable to a complex situation and deducible from general laws, though not yet so deduced. The very fact that a law is applicable to a class of complex facts or phenomena creates a presumption that it is deducible from

several higher or elementary laws. The laws of wind and rain, for example, are believed to be deducible from higher uniformities discussed in meteorology.

(2) An empirical law may express a relation between a remote antecedent and a remote consequent, passing over intermediate links. When, for example, it is said that 'a seed is the cause of a tree,' we have to do with an empirical law of such a description; for the seed can never become a tree without the help of intermediate conditions, such as planting, watering, *etc.*

(2) An empirical law expressing a relation between a remote antecedent and a remote consequent.

(3) An empirical law may express a relation among the co-effects of one and the same cause, whether such co-effects are related by way of (*a*) Succession or (*b*) Co-existence. (*a*) Succession is illustrated in the case of day and night and in the flow of the seasons. (*b*) Co-existence is illustrated in the case of the simultaneous effects produced in the different organs by a drug. Arsenic may produce purging, vomiting, *etc.*, simultaneously. A country engaged in war may similarly have its economy disturbed at once in various departments of its government.

(3) An empirical law expressing a relation among the co-effects of a cause, which may be connected either (*a*) by succession or (*b*) by co-existence.

It may be mentioned in this connection that the sciences which rest on empirical laws are generally less certain and progressive than those which employ the derivative ones. Thus, the science of medicine in its present condition is hardly beyond the empirical stage, as a knowledge of the effects of drugs is derived chiefly, if not wholly, from observation and experiment and not from higher laws connecting the properties of

Empirical sciences are more or less precarious.

remedies administered with the conditions of life or health. And hence any extension in the use of medicine to new circumstances (*e.g.*, men or animals of different constitutions, habits or countries) is more or less precarious.

Laws are useful (1) objectively and (2) subjectively. (1) Objectively, a law unites many facts together;

§ 5. Utility of Law and the Relative Usefulness of Its Different Forms. The usefulness of laws is illustrated both (1) objectively and (2) subjectively. (1) Objectively, a law connects diverse facts coming within its province: facts which would otherwise be disconnected and detached are thus reduced to a system by a law connecting them. The law of gravitation, for example, brings together all material bodies attracting each other, which otherwise would remain detached. (2) Subjectively, a law enables us to remember facts more easily than otherwise it would have been possible for us to do. It is not practicable for us to remember the numerous individual instances one by one; but these may be retained by reference to a law connecting them. Moreover, explanation always involves reference to laws: we explain phenomena when we refer them to their causes and indicate the laws by which such phenomena are brought about. (*Vide* Chap. XXIV, § 2.)

and (2) subjectively, it enables the mind to remember and explain facts.

All laws, however, are not of equal value.

Though laws generally are thus useful, yet their utility is not of the same character always. Some laws are more useful theoretically, while others are more useful practically. The Ultimate or Primary Laws are generally of greater theoretical value. Since the end of knowledge is unification, we approach to this end as we arrive at higher and

The ultimate or primary laws are of theoretical importance,

higher generalities. A very wide ultimate law, which can connect numerous facts, enables us to systematize knowledge pre-eminently. From the practical stand-point, however, the Secondary Laws are of greater importance: as these laws are in touch with facts, they enable us to solve practical problems more successfully than Ultimate Laws, which are of greater theoretical value. It is not of much consequence to a medical practitioner to be aware simply of the most general laws of health or of drugs. To be successful in practice, he must study the laws which govern the special form of the disease which he is ordinarily called upon to treat, and he should similarly study the special properties of the drugs which he ordinarily employs.

while the secondary laws are of greater practical value.

As a Secondary Law is applicable to a complex situation, we should be careful to extend such a law beyond the narrow limits of time, place, and circumstances where it has been found to be true. We should not, for example, extend to other nations the laws which are specially illustrated in our own constitution. If, however, we are disposed to extend the application of a Secondary Law beyond its known province, we must remember that such extension is justifiable more in the case of Derivative Laws than in the case of Empirical Laws. The rise of water in the pump, for example, up to the height of 33 feet can scarcely be confidently extended to other places or other liquids, if the law be viewed as merely Empirical; but when the law is regarded as Derivative (having

Secondary Laws should not be extended beyond their known sphere without great caution.

The extension of Derivative Laws to unknown cases is more certain than that of

Empirical Laws.

been deduced from atmospheric pressure), then we can confidently extend it to other similar situations, where such pressure remains unaltered.

§ 6. **Exercises.**

1. What is a Law? Distinguish a Law of the State, a Law of Nature, and a Logical Law, illustrating your meaning with examples. Science must assume that Nature is subject to Law: explain why it must do so.

2. What are the postulates of the Laws of Nature? Determine the character of the Law of the Uniformity of Nature.

3. Distinguish between (1) Axioms and Laws of Nature, (2) Primary and Secondary Laws, and indicate their relative importance in science and practice.

4. Distinguish between (1) Derivative and Empirical, and (2) Invariable and Approximate, Laws, and determine their relative values as conditions of proof.

5. Distinguish between Laws of Succession and Co-existence, and point out their different forms.

6. Distinguish between Laws and Facts, and estimate their relative importance in scientific inquiry.

7. Clearly explain what is implied in the conception of the World as a System of Laws.

8. What do you understand by the Laws of Nature? Do they rest on any primary assumption? How are such laws established? Explain and illustrate their different forms.

CHAPTER XXIV.

SCIENTIFIC EXPLANATION.

§ 1. **Character of Explanation.** Explanation (from Lat. *explano—ex*, out of, and *plano*, to make plain) implies, as the etymology of the word indicates, the act of making plain or intelligible what otherwise seems to be obscure or mysterious. Explanation thus presupposes a prior state of perplexity, which it tries to remove. A fact or phenomenon is explained when it is made clear to the understanding; and, the essential nature of our intelligence being assimilation and discrimination, things are made clear when their points of similarity and difference are shown. Thus, we understand what a pen or pencil is, when it is pointed out to us that it is an instrument for writing. Here we detect a similarity between a pen or pencil and what is called an instrument, and we find also that its distinguishing feature lies in being used for writing. When we come to know these points of similarity and difference, our curiosity is satisfied and the object becomes familiar to us.

The end of Explanation is to make clear to the understanding what is otherwise obscure.

As understanding consists in identifying and distinguishing things, the aim of Explanation is to indicate the points of similarity and difference.

Often the aspect of similarity is prominently illustrated in explanation, as when we explain a thing by simply referring it to its appropriate class (such as 'this is a pen' or 'that is a goat'). Classification may thus be regarded as a rudimentary form

Often the aspect of similarity is prominent in Explanation,

Classification is a form of explanation.

of explanation. And the reason is patent. Whenever we classify an object, we know its points of similarity and difference by reference to the characteristics of the class to which it is referred. But classification by itself illustrates the aspect of similarity in a marked degree; we bring an object under a class when we notice the striking points of similarity. Hence usually similarity or likeness is taken to be the ground of all explanation. "Our only progress from the obscure to the plain, from the mysterious to the intelligible," writes Bain, "is to find out *resemblances* among facts, to make different phenomena, as it were, fraternize" (*Induction*, p. 116.) Common explanation, however, is generally satisfied with the detection of the superficial points of likeness; it seldom tries to go deep and discover the deep-seated points of community. This is the aim of scientific explanation. (*Vide* Chap. XXV, § 2.)

Hence similarity is viewed as the ground of explanation.

Popular explanation is concerned with the superficial, while the scientific form, with the deep-seated, points of similarity.

§ 2. Popular and Scientific Explanation. To explain, as we have said, is to render a fact or phenomenon clear or intelligible; and, to render it clear or intelligible, it must be connected with prior knowledge, it must be likened to what is already familiar to us. Thus, explanation is always relative to the prior intellectual attainments of an individual. What serves as an explanation to a child may be of no value to an adult; what may satisfy a rustic or pagan may fail to convince a savant or Christian. That eclipses are produced by a dragon swallowing the sun or the moon, or that storms are produced by

Explanation is relative to prior attainments.

the wrath of Neptune or Jupiter may not seem strange to the ignorant, who know how rabbits are swallowed by serpents or how things are set in confusion by the wrath of individuals. But such explanations fail to convince one having an insight into the laws of Nature.

Explanation often consists in assigning the cause of the phenomenon to be explained.

We find also that facts or phenomena are explained when the agency or cause producing them is indicated. Thus, we explain a cold by reference to exposure to inclement weather, or we mention that the difficulty of a northern invasion of India lies in the presence of the Himalayas. And this mode of explanation is allied to the generalizing process indicated above. The cause being invariable in character enables us to assimilate all like effects. The single cause establishes a sort of unity among its effects which are thus connected by a common bond. The cause which explains this effect, explains all like effects. The cause is thus regarded as the common source of all such effects. When, therefore, we explain a phenomenon by reference to its cause, we mean to say that all similar phenomena may similarly be explained. Thus, in assigning a cause, we generalize and assimilate facts.

Cause is a means of generalization.

We have read that the difference between ordinary every-day knowledge and science lies in the fact that, in the former, we are interested in individual instances or peculiarities, while, in the latter, in common features or general characteristics. (*Vide* Chap. I, § 7.) And this difference is illustrated also in the case of Explanation. In the

Popular explanation aims at discovering the special cause in any case: while scientific explanation aims at discovering the general conditions.

ordinary affairs of life, we want to know what throws light on this or that circumstance, what can remove this or that difficulty or obstacle. Science, however, is concerned with the explanation of facts in general, with the discovery of general laws or grounds which elucidate this and all similar cases. "There is," says Bain, "a special and every-day form of explanation that consists in assigning the agency in a particular occurrence; as when we ask—what stops the way? Who wrote Junius? Who discovered gunpowder? These questions belong to our practical wants and urgencies, but the answer does not involve the process of scientific explanation. If, however, we proceed from the 'who' or 'what' to the '*why*':—why does A's carriage stop the way? why did the author of Junius write so bitterly?—there is an opening for the higher scientific process." (*Induction*, p. 116.)

Explanation is either (1) of a particular event or (2) of a general law.

Explanation is either (1) of a (particular) fact or (2) of a (general) law. Popular explanation is concerned chiefly with the former, while scientific explanation, mostly with the latter. And, (1) even in the *explanation of a fact*, the popular form refers, as mentioned above, to some special circumstance which brings immediate practical gain, while the scientific form refers to general conditions or laws which tend to enlarge our theoretical knowledge and thus to contribute to our future advantage.

Scientific explanation traces (1) particular events to their general conditions

and (2) general laws to higher laws, known or supposed.

(2) The scientific *explanation of a law* ordinarily consists in tracing it to some higher law, real or supposed. Explanation in this case consists in deducing the law to be explained from some other

known law or from some hypothesis which is expected to throw light on it. Thus, magnetism is traced to electric polarity or solution to heterogeneous molecular attraction. Different laws may likewise be connected by similarity and referred to some higher law, as when combustion and metallic corrosion are viewed as but different forms of oxidation. Scientific explanation, accordingly, consists, as Mr. Read points out, in "discovering, deducing, and assimilating the laws of phenomena." (*Logic*, p. 276.) We should remember in this connection that Explanation and Hypothesis are very closely related: the general end of hypotheses is explanation; and explanation, consequently, often involves a reference to hypothesis. "Explanation, in the scientific sense", observes Fowler, "means the reduction of a series of facts which occur uniformly but are not connected by any known law of causation into a series which is so connected, or the reduction of complex laws of causation into simpler laws. If no such laws of causation are known to exist, we may *suppose* or *imagine* a law that would fulfil the requirement; and this *supposed* law would be a hypothesis." (*Induction*, p. 92.)

Mr. Read's definition of Explanation.

Explanation and Hypothesis are closely connected.

§ **3. Forms of Scientific Explanation.** Mill mentions three forms of scientific explanation:—

Three Forms of Scientific Explanation:

(1) *Analysis.* When a joint or complex effect is referred to the laws of its conditions or causes; for example, when the path of a projectile is explained by reference to the laws of gravitation, initial force, and resistance of the air.

(1) A joint effect is explained by reference to elementary laws.

(2) A remote effect is explained by reference to intermediate agencies.

(2) *Concatenation.* When a remote effect is explained by reference to the intermediate agencies or links; for example, when a good crop is explained by reference to favourable weather and the industry of the husbandman. Likewise, in regarding sea water as the cause of rain, we have to supply the intermediate links (such as evaporation, condensation, electric discharge, etc.) to explain the connection. Proverbs, as pithy sayings, generally pass over intermediate steps (*e.g.*, 'No pains, no gains'). To explain them, therefore, we must unfold these steps.

(3) A lower or less general law is explained by reference to a higher or wider law.

(3) *Subsumption.* The subsumption or inclusion of inferior laws under higher laws; when, for example, terrestrial gravity is explained by reference to the law of universal gravitation. Similarly, the minor laws of antithesis in rhetoric, contrast in works of art, novelty in attention, and variety in agreeable experience are all explained by the fundamental law of relativity.

The essence of scientific explanation lies in discovering a causal connection.

We must bear in mind that the essence of scientific explanation in every case lies in discovering a causal connection, which, as we have seen, is a sure means of generalization. The more we can connect one fact or law with others, the greater the relief to our understanding, and consequently the more satisfactory is our explanation. And in this we are materially aided by causation. "Not any sort of likeness," says Mr. Read, "suffices for scientific explanation: it must be 'fundamental' or (as this is a vague expression) we may say that the only satisfactory explanation of concrete things

or events, is to discover their likeness to others in respect of Causation." (*Logic*, p. 281.)

§ 4. **Limits of Explanation.** It is apparent from the preceding remarks that the limits of explanation are the limits to assimilation. When one thing or phenomenon cannot be referred to a law or other similar phenomena, it remains unexplained. Colour, for example, cannot be likened to anything else; and so any attempt to explain it must prove futile. Once scarlet colour was explained to a blind man as very loud; and he exclaimed, 'yes, it is as loud as the beat of a drum.' It was quite natural, for the blind man could interpret 'loud' only by reference to his experience of sound.

The limits of explanation are the limits to assimilation.

On a careful examination we find that it is not possible for us to explain (1) elementary sensations (*e. g.*, colour, taste, smell), (2) the ultimate forces or properties of matter (*e. g.*, extension, inertia, gravity), and (3) individual peculiarities of concrete objects. These cannot be assimilated: a colour, for example, can never be likened to a sound or smell, nor physical energy to chemical affinity, nor individual peculiarities to specific qualities or properties. It is evident from this that the ultimate laws of nature and the elementary experiences of the mind can never be explained. We should remember in this connection another limit to explanation, which arises from its very nature. Explanation must always be relative: it can never be absolute. To explain one fact we must fall back upon another; and so on we may proceed;

We cannot explain (1) elementary sensations, (2) ultimate forces or properties, (3) individual peculiarities,

and (4) axioms or ultimate principles.

but finally there must be a halting place. (4) Complete explanation is thus unattainable so far as the ultimate principles are concerned: they must be assumed as they are, without any further attempt at tracing them to higher principles still. "The principles of Contradiction, Mediate Equality and Causation", says Mr. Read, "remain incapable of subsumption; nor can any one of them be reduced to another; so that they remain unexplained." (*Logic*, p. 283.)

Illusory explanations, are only apparent explanations, the principal forms of which are—

§ 5. **Illusory Explanations.** Illusory explanations are those which are of a superficial character,—which pretend to explain facts or phenomena without really doing so. The principal forms of such explanation may be indicated thus—

(1) To explain a fact superficially by reference to what is familiar;

(1) Often we explain a fact by reference to something with which we are familiar, the points of similarity being but superficial. For example, thunderbolts are explained as shafts of fire hurled by Jove; and solitary boulders, as missiles flung by gaints.

(2) merely to vary the expression;

(2) Again, we sometimes state the same thing in a different form of language and this is a fruitful source of erroneous explanations. Moliere's physician, for example, explains the sleep-producing property of opium by reference to its dormitive power; and we similarly explain a slip of the feet by reference to the slippery character of the ground.

(3) to explain an ultimate fact.

(3) Sometimes we are not satisfied with the best explanation offered, and so we push our inquiry farther, until we explain the clear by the obscure. Thus, Newton was not satisfied with

gravity as an ultimate fact explaining the attraction of bodies to one another. He could not imagine how one lump of matter could act on another at a distance ; and he, accordingly, longed to discover some fluid medium through which gravity might be supposed to act. But the law of gravitation may be regarded as the final explanation of falling bodies. Any supposed medium only tends to mystify what is otherwise clear.

To suppose a familiar fact as clear and simple is also fallacious.

It may be mentioned in this connection that at times we fall into the opposite mistake of supposing even complex facts or phenomena as simple because they are familiar. And hence we are disposed to treat them as intelligible in themselves, without any reference to anything else. Thus, combustion or the succession of day and night may seem to be such a familiar fact as to require no explanation. But, surely familiarity is no test of simplicity or intelligibility.

§ 6. Exercises.

1. Determine the character of Explanation. Distinguish between Popular and Scientific Explanation.

2. 'To explain a phenomenon is to assign its cause.' How ?

3. Describe and illustrate the different forms of Scientific Explanation.

4. Point out the limits of Scientific Explanation. Can we be certain that any scientific explanation is complete and final ?

5. How does Hypothesis lead on to Explanation ? How is Explanation related to Induction ?

6. Distinguish between Genuine and Illusory Explanations, indicating the chief forms of the latter.

7. Wherein does Explanation differ from Proof? Does everything admit of explanation? If not, where does explanation cease?

8. Illustrate the several ways in which facts and generalizations from facts may be explained. Are all modes o Scientific Explanation reducible to one principle?

9. Elucidate—'The object of Science is explanation.'

BOOK IV.

ACCESSORIES OF INFERENCE.

CHAPTER XXV.

DEFINITION.

§ 1. **Preliminary.** Having considered the different forms of Inference, let us now proceed to study the logical processes which are its accessories. Logical processes being processes of the human mind are all interconnected: they act and re-act on one another. (*Vide* Chap. XXXI, § 1.) Thus, Inferences, as we have seen, involve Propositions; and Propositions, Terms. (*Vide* Chap. IV, § 1.) Again, Terms or Names, to be of any use, must carry some sense, *i.e.*, must involve a reference to Definition and Classification. To name an object is to refer it to a class and to indicate what is implied by it. It appears, then, that Definition, Classification, and Naming are all more or less pre-supposed in every form of Inference. But, looked at from a different point of view, it would seem that Inference determines all these processes. We gather the meanings of Names through Classification and Inference. If we exclude proper names, which have very little logical value, we find that Names ordinarily involve a general reference—indicating a passage from the known to the unknown. Thus, the meanings of Names grow fuller and fuller by successive inferences. It is in this way that a child comes to

Logical processes are all interconnected.

While Definition, Classification, and Naming ensure the validity of Inference, Inference in its turn secures the correctness of these processes.

know, for example, that cats mew, have whiskers, four legs, a tail, a peculiar form, *etc.** Noticing these features in some cats, a child is led to think that they are present in all cats. The meaning of the term 'cat' is determined by a series of inferences. But a name, in its connotative aspect, implies definition and, in its denotative aspect, implies classification. Inference may, accordingly, be said to underlie the processes of Definition, Classification, and Naming.

We see, then, that Inference, Definition, Classification, and Naming are all closely connected. Valid inference ensures the correctness of Definition, Classification, and Naming; and the correctness of these processes in its turn secures the validity of Inference. Though, however, these processes thus interact, we are not concerned in Logic with this interaction. We have nothing to do here with the mental processes themselves; we are concerned only with the mental products. (*Vide* Chap. III, § 2.) And, since we have considered Inferences above, we shall now turn our attention to the exposition of their accessories. And the reason for considering these after Inference is that, as thought-products, they are often determined by prior inferences.

Definition, Classification, and Naming, as products of thought, are thus often determined by prior Inferences.

§ 2. Character and Limits of Definition. Definition is but a compendious form of

Definition is

* Naming is really a complex process, involving not merely inference but also the influence of the social intelligence. We are, however, concerned here not with the psychological history of naming or conception, but with the logical product. (*Vide* Chap. IV, § 3, foot-note.)

explanation. Its aim is to unfold the meaning of a term or class as succinctly as possible. A fact, however, is intelligible to us when it is assimilated and discriminated. (*Vide* Chap. XXIV, § 1.) To know, for example, what a rose is, we must distinguish it from other things and identify it with the members of its own class. It is thus known to be a flower having a peculiar fragrance, shape, and structure. But, as we have read in the last chapter, explanation may be either superficial or genuine. The popular form refers merely to the obvious features, whether they are essential to the thing explained or not. In the case of Definition, likewise, the popular form may be content with the bare delineation of the outward and accidental features, as when man is taken to be a laughing animal. This is what is known as **Description.** Definition proper is allied to scientific explanation and, like it, is concerned with the determination of the essential qualities of the thing defined. (*Vide* Chap. I, § 6 and Chap. XXIV, § 2.) And the general condition of a valid definition is that it must indicate the fundamental points of similarity and difference; or, as the scholastic formula goes, it must be *per genus et differentiam* (the genus being the point of similarity and the differentiam, the point of difference). The accidental and superficial features, however successful in the ordinary affairs of life in indicating a class, generally fail to give an accurate and precise knowledge of it. It is said that Plato defined 'man' as 'a featherless biped.' And Diogenes, to

a compendious form of explanation.

It thus involves assimilation and discrimination.

In Description only the accidental features are mentioned;

while in Definition, the essential qualities.

Definition should be *per genus et differentia.*

expose the fallacy of such a definition, plucked a fowl and introduced it into his school as 'Plato's man.' Plato, of course, was driven to modify his definition by adding the further difference 'with broad nails.' But such a procedure is always more or less precarious. Hence, in defining a class, the fundamental features alone should be given. In Description there is rather an appeal to imagination, while in Definition, an appeal to thought. It is clear from these remarks that those terms which have no connotation cannot be defined; but they can be described. Thus, all definable terms can be described; but all terms which can be described cannot be defined.

Description appeals to imagination, while Definition to reason.

It may be mentioned here that Explanation, like Definition, may be viewed as but a stage in advance of Description. If in Description we colligate or unite facts by reference to the features or characters which lie on the surface, in Explanation, as in Definition, we go deeper and try to discover laws or conditions which throw light on these features or characters. In the one case we are concerned rather with the 'what' of things, while in the other we consider the 'why.' Thus, when Kepler surmised from the observation of a few positions of the planet Mars that its orbit lay in an ellipse and he also found that the orbits of the other planets were of a similar character, he supplied merely a descriptive hypothesis, and this was subsequently converted into an explanatory theory by Newton when he showed that the character of such an orbit in all these cases followed

Explanation, like Definition, indicates a stage in advance of Description.

from the universal law of gravitation binding together all material bodies moving in space. If Description, therefore, may be said to rest on connections which furnish a basis for empirical laws, Definition or Explanation may be said to appeal to characters which afford a ground for derivative laws.

Features of Definition : (1) Definition is always of a class and never of an individual.

Three important conditions follow from the above account of Definition. (1) Definition must be always of a class and never of an individual. The distinctive character of an individual is found in certain peculiarities not to be found in others. The group of peculiarities constituting, for example, John can never be found in Jones or James. Hence it can never be likened to any other group; and so it cannot be defined. We may, no doubt, compare some of the qualities (*e.g.*, red hair, tall stature, aquiline nose, bald head) with the qualities of other persons; but such comparison would mean that each quality, viewed by itself, is general, being illustrated in several persons. But the entire aggregate constituting what we call 'John,' can never be likened to any other aggregate. In fact, to suppose it as possible is to overlook the individuality of John. (2) Abstract terms are more easily defined than concrete terms, provided the abstract terms do not express elementary qualities, which cannot be likened to others. And its reason is found in the fact that a definition unfolds the connotation of a term. The connotative aspect is more prominently present in the case of abstract terms, while the denotative

(2) Abstract terms are more easily defined.

aspect, in the case of concrete terms. In defining a concrete term, accordingly, we have to withdraw our attention from the denotative, and concentrate it on the connotative, aspect. No such abstraction is, however, necessary in defining an abstract term.

Attributives, however, cannot be defined.

Attributives, however, cannot be defined, since they express qualities only when standing as predicates; but they may be defined through the corresponding abstract terms. Thus, though we cannot define 'virtuous,' we may define 'virtue' as 'the excellence of character acquired by habitual obedience to moral law.' Similarly, to define 'the virtuous' would be to define a concrete term and not an attributive. [*Vide* Chap. V, § 8.]

(3) Only the fundamental qualities, constituting connotation, should be included in the definition.

(3) It is also evident from what has been said above that a definition should include only the fundamental qualities as otherwise it would be contrary to its purpose. As a definition is a condensed form of explanation, it must avoid prolixity and redundancy alike. The qualities mentioned in a definition should be such as would enable us to comprehend the thing defined in the shortest and easiest way possible. These should, accordingly, be the cardinal or fundamental qualities on which the minor qualities hang and from which these can easily be deduced.

***Limits of Definition*:**

(1) Elementary qualities cannot be defined.

From the preceding remarks we can easily determine the limits of Definition. The limits are :—(1) We cannot define elementary qualities, such as pleasure or pain, sweetness or bitterness, redness or greenness, likeness or unlikeness. And the reason is found in the fact that they represent

unique features which cannot be likened to other things: we must either know them directly or not know them at all. If Nature has given us the power to apprehend these features, no definition is necessary; but if Nature be unkind in this respect, no definition is adequate. (2) From this it follows that the highest or summum genus cannot be defined. 'Thing,' for example, expresses pure being, which can never be viewed as resembling anything else. (3) Individual objects, as explained above, are also incapable of definition. We have read that proper names are devoid of connotation (*Vide* Chap. V, § 11); and hence it is not possible to define them.

(2) Summum genus cannot be defined.

(3) Individuals cannot be defined.

§ 3. **Forms of Definition.** Definitions have been distinguished differently from different stand-points. Let us notice here some of the distinctions.

(1) The most prominent distinction is that between Inductive and Deductive Definitions. In **Inductive Definition** we try to determine the meaning of a class by examining its particular instances. Like inductive inference it proceeds by observation and generalization. To arrive at the definition of 'student' or 'book,' for example, we have to observe different individual students or books and thus to gather their essential qualities which go to fix the connotation. In **Deductive Definition**, on the other hand, we explain the meaning of a complex notion by analysing it into its constituent simpler notions, as when we define a triangle to be a figure bounded by three sides. Like deductive inference,

In *Inductive Definition*, the connotation is determined by an examination of facts;

while in *Deductive Definition*, the accepted connotation is simply unfolded.

it assumes its data as having a fixed connotation and sets before itself the task of merely unfolding it. We shall advert to the conditions of these two forms of Definition in the two following sections; but we should remember that both of them are characterized by the same feature of noting the important points of similarity and difference.

A *Real Definition* explains the meaning of an existing class, while a *Nominal Definition* explains only the meaning of a name.

(2) Definitions have also been distinguished into **Real and Nominal.** A definition is said to be real when it explains the meaning of an actually existing thing; while it is viewed as nominal or verbal when it merely unfolds the meaning of a name or term, without any reference to the actual existence of the corresponding objects. And, as in both the cases we have to do with ideas or notions, the one may be viewed as clearing up the relation of ideas to things, while the other, of names to notions. (*Vide* Hamilton's *Reid*, p. 691.) A question is raised at times in this connection—Whether a definition is directly concerned with things or with names? It is evident that the need of a definition is felt only when we have passed the stage of infancy and are constrained to think only by means of names. (*Vide* Chap. I, § 5.) Thus, "All definitions are of names, and of names only; but, in some definitions, it is clearly apparent, that nothing is intended except to explain the meaning of the word; while in others, besides explaining the meaning of the word, it is intended to be implied that there exists a thing, corresponding to the word." (Mill, *Logic*, I, p. 162.) Mansel contends that, as in every definition we have to do with ideas or notions,

Logic is concerned with "notional definitions only." "Definition," he writes, "is confined to the analysis and separate exposition of the attributes contained in a given concept, and determines not their reality but their conceivability." (*Prolegomena Logica*, pp. 202, 204.) However suitable such a view might have been, when formal truth was regarded as the sole end of logical inquiry, it can scarcely be maintained now-a-days when material truth is so prominently made the end of Inductive Logic. But, it is nevertheless true that Logic is directly concerned with notions or thoughts: "Every definition," says Ueberweg, "defines not the name, nor the thing, but the *notion*, and with it the name and the thing so far as this is possible." (*Logic*, p. 167.) It may be said, however, in defence of the nominalistic position here, that there are three principal reasons which seem to justify the view that definitions are primarily concerned with names: (1) we invariably think by means of names when we are in a position to employ or understand definitions; (2) definitions themselves would be vague and indefinite unless couched in precise forms of expression; and (3) the end of definition in every case seems to be to fix an accurate use of terms. (Cf. the Socratic art of Definition.) Definition, as Whately observes, "is used in Logic to signify an expression which explains any term, so as to *separate* it from everything else, as a boundary separates fields." (*Logic*, p. 94.) And this is quite consonant with the etymological sense of the term, which implies 'laying down a boundary' or marking out

Definitions are primarily concerned with names.

Reasons for this view.

the limits in any case (Lat. *definio*—*de*, down, and *finio*, to limit, from *finis*, end).

A Substantial Definition explains the essence or connotation of a notion, while a *Genetic Definition* enables us to arrive at an idea of it by indicating the way in which it is formed.

(3) Definitions have further been distinguished into **Substantial** and **Genetic**. A definition is said to be substantial when it unfolds the connotation or essential qualities of the notion defined; while it is taken as genetic, when it indicates the way by which we can arrive at an idea of it. Thus, the definition of 'triangle' as a three-sided rectilineal figure is a substantial definition; but the definition that it is formed by a perpendicular plane passing through the apex of a cone is a genetic definition. Similarly, the definition that sensation is an elementary mental phenomenon produced by the stimulation of the peripheral extremity of a sensory nerve, when the current is carried to the brain, may be regarded as a genetic definition of sensation, though there can be no substantial definition of it owing to its elementary character. Substantial or Essential Definitions are thus definitions proper, while Genetic Definitions merely enable us to have an idea of a thing by indicating the mode of its genesis or formation.

Material conditions determine Inductive Definition.

§ 4. **Material Conditions of Definition.** The material conditions of definition regulate Inductive Definition. To arrive at a correct definition of a term, we must ascertain the essential qualities of the class indicated by it. We indifferently speak of defining things and of defining names, because names stand for things; and any attempt at determining the true meaning of a term must have reference to the things signified

by it. "Definition," says Mr. Stock, "is of things through names." (*Logic*, p. 114.) We have seen that definition must always be of a class and never of an individual. (*Vide* § 2.) It is never possible for us to enumerate exhaustively all the qualities of an individual, which mark him out from the other members of the same species. To attempt to do so would be to exhaust infinity, which evidently can never be done. Hence definition is restricted to general terms alone. And, to gather the true sense of such terms, we must carefully examine the individuals constituting a class with a view to determine their common and essential features. This we can do only by noting, as explained above, the important points of similarity and difference. To arrive at a true definition, we must, therefore, observe the following rules :—

A true definition must be based on an examination of facts.

(1) *We should bring together individuals indicated by the term to be defined as well as those indicated by opposite or contrary terms.* It is not meant by this that we are to assemble for comparison all the individuals of a class, which is never possible. It implies merely that we should select for comparison the representative members of the class to be defined as well as those of the opposite classes. This will enable us to discover not only the important points of similarity but also the striking points of difference. This is what is called by Bain the positive and the negative method of definition. They are really parts of one process, which aims at discovering the true import of a class or term. Bain well observes, "As the state-

(1) We should examine the individuals of the class to be defined as well as those of the opposite classes to ascertain the essential points of similarity and difference.

Representative instances should be examined in every case.

ment of what is common to a number of particular things, Definition is essentially a process of generalization; while neither particular things, nor their agreements, have any distinct meaning, unless there be assignable a distinct opposite. The act of Defining, therefore, consists of a generalizing operation, rendered precise at every step by explicit or implicit opposition, negation or contrast." (*Induction*, p. 155.) Thus, to define 'Matter,' 'Solid,' 'Metal,' or 'Food' we are to examine representative examples of these classes as well as those of the opposite classes—such as 'Mind,' 'Liquid,' 'Non-metals,' and 'Poison' as well as 'Stimulants'—and thereby to find out the important points of similarity and difference to be included in the definition.

This implies the employment of the positive and the negative method.

(2) We should include only the fundamental qualities in a definition.

(2) *We should include in the definition only the fundamental qualities from which many other important qualities follow.* The reason of this rule is evident from the very nature of definition, which aims at conveying correct information of a class in the briefest form possible. Thus, though there are several points of similarity and difference in the case of 'Matter' or 'Solid,' we include only the fundamental or essential qualities (such as 'extension' and 'inertia' in the one case, and 'resistance to force applied to change the form' in the other) in the definition.

(3) We should be guided by the knowledge supplied by the sciences.

(3) We should take into account the knowledge supplied by the sciences in framing our definitions. The connotation of a term is determined, as we have seen, not by popular usage but by scientific

investigation. (*Vide* Chap. IV, § 4.) A definition resting merely on popular estimate is often variable and precarious. We can render our definitions comparatively stable and accurate only when we include in them the qualities proved to be fundamental by scientific research. The **golden rule of definition**, accordingly, is *to take into account the most important and the most numerous points of community among the objects constituting the class to be defined.* This, as we shall see, is also the golden rule of Classification. (*Vide* Chap. XXVI, § 2.)

Golden rule of definition.

It is contended by some writers that the Inductive Definition is rather an impracticable process, since it is by no means an easy task to determine the qualities which are essential and common to all the members of a class. Difficulties are felt here in two ways :—(1) The number of individuals constituting a class is too large to be viewed together. Even the number of representative instances is at times numerous and various (as in the case of animals or plants), which can never be adequately considered for a correct definition. (2) There are some doubtful instances which may as well be brought under one class as under its opposite. They may be called *marginal instances,* lying in the middle, which may be drawn either to this side or to that. Is, for example, ether material or immaterial ? Is jelly solid or liquid ? Is dawn or evening day or night ? Is arsenic or tellurium a metal or a non-metal ? Is sponge a plant or an animal ? As it is difficult to decide such cases by hard and fast definitions—which, it

Inductive Definition is pronounced by some writers as impracticable.

Alleged difficulties :

(1) Individuals are too numerous to be adequately examined ; and

(2) there are marginal instances which cannot appropriately be included in a definition.

is urged, would be more or less arbitrary—a form of definition, called **Definition by Type**, is suggested by the advocates of this view as more suitable to the practical needs of life. Such a definition consists in referring to a representative member or variety of the class to be defined, as conspicuously exhibiting its prominent features, and thus rendering it intelligible to us. (*Vide* Chap. XXVI, § 3.) "The type-species of every genus, the type-genus of every family," says Whewell, "is that one which possesses all the characters and properties of the genus in a marked and prominent manner." (*History of the Inductive Sciences*, II, p. 122.)

Hence *Definition by Type* is suggested by the advocates of this view.

It may be said, however, in defence of the Inductive Definition that (1) it is not necessary to examine all the members of a class in order to discover its essential attributes which are to be embodied in its definition. An examination of representative instances and at times a careful and exhaustive analysis of even a single instance may enable us to find out such attributes. (*Vide* Chap. XVIII, § 2 and Chap. XXII § 6.) Again, (2) the marginal instances do not invalidate a definition, since they are but exceptions to the rule. No one ever confounds opposite classes because he cannot precisely determine the character of an instance lying between them. "A certain *margin*," says Bain, "must be allowed as *indetermined*, and as open to difference of opinion; and such a margin of ambiguity is not to be held as invalidating the radical contrast of qualities on either side. No one would enter into a dispute as to the moment

But the above objection to Inductive Definition is untenable for the following reasons:

(1) An examination of representative instances alone may enable us to ascertain the essential attributes.

(2) The marginal instances are rather exceptions,

when day passed into night; nor would the uncertainty as to this moment be admitted as a reason for confounding day and night. We must agree to differ upon the instants of transition in all such cases. While the great body of the non-metals can be distinctly marked off from the metals, we refrain from positively maintaining arsenic and tellurium to be of either class; they are transition individuals, the 'frontier' instances of Bacon; in that position we leave them." [*Induction*, pp. 160-161.] (3) Moreover, the so-called Type is viewed as a type because it illustrates in it the common and essential attributes of a class. The peculiarities of a group considered as a type are not taken into account; only those qualities are considered which are generally found among the members of the class defined. Thus, the determination of a type involves a reference to definition. If we be disposed to take into account the marginal instances, then, instead of denying the possibility of Inductive Definition and substituting for it Definition by Type, it would be more reasonable to maintain that Inductive Definitions are rather approximate in character, being applicable to *most* members of the class defined. Moreover, there is greater possibility for a variation of Type than for that of Definition. "An Approximate Definition," observes Mr. Read, "is less misleading than the indication of a Type; for the latter method seems to imply that the group which is now typical has a greater permanence or reality than its co-ordinate groups; whereas, for aught we know, one of the outside

(3) Type in any case involves Definition.

Even if marginal instances be taken into account, it would be more reasonable to maintain that Inductive Definition is of an approximate character than to deny its possibility altogether.

Moreover Types are more variable than Definitions.

varieties or species may even now be superseding and extinguishing it. But the statement of a definition as approximate, is an honest confession that both the definition and classification are (like a provisional hypothesis) merely the best account we can give of the matter according to our present knowledge." (*Logic*, p. 326.)

The difference between Definition and Induction lies in the fact that the conjunction of qualities is assumed in the former, while it has to be proved in the latter. Definition embodies the results of Induction.

We may mention in this connection that the difference between Inductive Definition and Induction proper lies in the fact that the connection of qualities characterizing a class is taken for granted in the former, while it has to be proved in the latter. "In definition," writes Bain, "the conjunction is tacitly assumed; in induction, it is laid open to question; it has to be *proved* or *disproved*." (*Induction*, p. 2.) In fact, Definition may be viewed as embodying the important results of Induction. (*Vide* Chap. XV, § 2.)

Formal conditions determine Deductive Definition.

§ 5. Formal Conditions of Definition. The formal conditions of definition regulate Deductive Definition. As in deductive inference we assume the data to be true, without inquiring into their material validity, so in deductive definition we assume terms as having a fixed connotation which we merely try to explain. (*Vide* Chap. IV, § 4.) The formal conditions have thus reference to the way in which we should unfold the central meaning of a notion and the way in which we are to express a definition in language that we may impart a correct information of the term defined. If Inductive Definition aims at determining the true meaning of a term by reference

They secure consistency and precision of thought and expression.

to the attributes of the class denoted by it, Deductive Definition aims at properly unfolding the connotation already ascertained and stating it in a clear and accurate form of expression. The formal conditions of definition are :—

(1) *A definition must set forth the entire connotation of the term defined.* In defining the term 'man,' for example, we should state its whole connotation—'animality' and 'rationality.' The scholastic formula of definition—that it should be *per genus et differentiam*—indicates the same necessity, for the genus and differentia constitute the connotation of a term. (*Vide* Chap. IV, § 5.) The violation of this rule gives rise to the fallacies of (*a*) incomplete, (*b*) redundant, and (*c*) accidental definitions. (*a*) When less than the entire connotation is stated, the definition is said to be *partial* or *incomplete.* If, instead of defining 'man' as a 'rational animal,' we simply state that 'man is an animal,' or 'man is rational,' then the definition becomes incomplete or imperfect. Similarly, if 'circle' be defined as 'a figure bounded by one line,' the definition is not adequate or complete, as part of its connotation is left out. (*b*) When more than the connotation is given in any case, the definition is said to be *overcomplete* or *redundant,* as when we define 'man' to be 'a rational animal capable of drawing inferences,' or we define 'triangle' to be 'a three-sided rectilineal figure, any two sides of which are together greater than the third.' (*c*) A definition is said to be *accidental* when, instead of stating the

(1) A definition should state the entire connotation of the term defined.

It should be *per genus et differentia.*

Violations of the rule :

(*a*) If part of the connotation be stated, a definition is incomplete ; and

(*b*) if more, it is redundant.

(*c*) If accidents be given,

there is Description.

connotation, it merely enumerates some accidental features, as when 'man' is said to be 'a laughing biped,' or 'love' is said to be 'a medley of endearments, jars, suspicions, reconcilements, wars, *etc.*, then peace again.' *Accidental Definitions* are but *Descriptions.* It is apparent from these remarks that an essential or strictly logical definition should not include in it a proprium or accident of the term defined.

Singular terms can only be described; but designations may be defined.

It follows from this rule that singular terms, having no connotation, cannot be defined, though they may be described. Designations, however, as significant singular terms, may be defined through the common terms entering into their composition. (*Vide* Chap. V, § 11.) Thus, the present Viceroy of India may be defined as the person who governs India now with regal authority as the representative of the King-Emperor. The fact is that designations are not essentially singular: they are singular only by accident.

(2) A definition must be convertible in extent with the denotation of the term defined.

(2) *The extent or denotation of a definition should be co-extensive with the extent or denotation of the term defined.* If this be not the case, then a definition either includes things not covered by the term or excludes those that are covered by it. This rule merely indicates that the connotation stated in the definition must be possessed by every individual denoted by the term. The violation of this rule gives rise to the fallacy of either *too wide* or *too narrow definition.* For example, the definition of 'an acute-angled triangle' as 'a three-sided figure having an acute angle' is too wide, while the definition

Violation of this rule gives rise to too wide or too

of 'a triangle' as 'a rectilineal figure of three equal sides' is too narrow. Similarly, the definition of 'table' as 'a material body' is too wide, while the definition of 'man' as 'a civilized rational animal' is too narrow.

narrow definitions.

(3) *A definition should not include the term defined or any of its synonyms.* To define a term by itself or by a synonymous expression is evidently to frustrate the end of definition as a satisfactory explanation of the term. The violation of this rule gives rise to the fallacy of *circle in definition.* When, for example, we define 'man' as 'a human being', 'mind' as 'a mental entity' or 'liquid' as 'a fluid substance', we commit such a fallacy. Such definitions are evidently quite useless: one, not knowing the meaning of the term defined, can never understand the meaning of such a definition.

(3) A definition should not be synonymous.

Violation of this rule gives rise to circle in definition.

(4) *A definition should be expressed in as clear and definite a form as possible.* The violation of this rule gives rise to the fault of *obscure* or *figurative definition.* Definitions expressed in obscure and figurative language are unintelligible and even liable to misconstruction. Such definitions should, therefore, be always avoided. Fallacies arising from the transgression of this rule may be illustrated by the following examples:—'Youth is the spring of life'; 'camel is the ship of the desert'; 'love is a sickness full of woes'; or love is

"The silver link, the silken tie,
Which heart to heart, and mind to mind,
In body and in soul can bind."

(4) A definition should be clear and not metaphorical.

Violation of this rule gives rise to obscure or figurative definition.

(5) *Negative definitions should be avoided, when-*

(5) A definition should

not be negative.

ever practicable. A negative definition by merely indicating difference fails to convey an adequate idea of the thing defined. A definition should, therefore, be never negative unless the whole meaning of the term defined is negative. The violation of this rule gives rise to the fault of *negative definition.* We commit a fallacy of this sort when we define 'birds' as 'not insects,' 'compound' as 'not an element,' or 'cold' as 'not hot.' Such definitions do not explain the meanings of the terms defined, and so they are useless.

Violation of this rule gives rise to negative definitions.

The first two rules refer to the contents of a definition; and the last three, to its expression.

It may be mentioned here that the first two of the above rules refer specially to the contents or meaning of a definition ; while the last three, rather to its expression.

Inductive and Deductive Definitions are complementary processes, securing the correctness and precision of notions and expressions.

§ 6. **Value of Definition**. The Inductive and Deductive Definitions are really complementary processes, securing the correct sense and proper use of terms. By the material conditions we secure the real truth of our definitions and by the formal conditions we ensure their due apprehension and application. Jointly they enable us to acquire truth and think clearly and consistently. Hence we find that the fundamental notions of every science are generally defined at the outset. Such definitions prevent misconception and confusion afterwards. We derive the following advantages from Definition—

Uses of definition :

(1) It renders the meanings of terms clear and definite.

(1) It renders the meaning of the term defined clear and definite, embodying it in language.

(2) It prevents tautology and inconsistency.

(2) It enables us to avoid tautology and contradiction. Repetition or inconsistency is often due

to the want of the proper comprehension of the meanings of the terms employed.

(3) It enables us to distinguish between real and verbal propositions and thus to detect truisms, which not infrequently pass for maxims of wisdom. How frequently do we not hear dicta like these—'It is just to give every man his due,' 'It is but proper that we should not reject reliable testimony,' 'The greater good should be preferred to the less'! These seem to convey useful information because we overlook the definitions of the terms used in them.

(3) It enables us to discern in any case whether an information is really new or not.

(4) It secures correct inference. Fallacious reasonings may often be traced to ambiguity of expression or to errors connected with the meanings of terms. (*Vide* Chap. XXX, § 5.)

(4) It is an aid to correct reasoning.

§ 7. Hints for Working out Exercises. (1) In determining whether a definition is materially correct, we should examine representative instances of the class defined and see whether they possess the qualities included in the definition.

(2) We should always try to define a term by reference to its proximate genus and differentia. We should always take care to see that no part of the differentia is left out, thereby rendering a definition incomplete.

(3) We should not include a proprium or an accident in a definition, which should always be concise, clear, and accurate.

(4) In distinguishing between Definition and Description, we should ascertain whether the fundamental qualities constituting the connotation have been given or only the superficial qualities appealing to imagination have been enumerated.

(5) We should remember that singular terms can only be described, though designations may be defined.

(6) Ambiguous terms should be defined by reference to the different senses in which they are used.

Illustrations.

1. Define Student, Library, Pen, Progress, Eloquence.

Student: A student is a person bent on the acquisition of knowledge.

Library: A Library is a collection of books belonging to a person or institution for the promotion of learning.

Pen: A pen is an instrument for writing by means of a fluid ink.

Progress: Progress is advance in a definite course or pursuit.

Eloquence: Eloquence is the influencing of men's feelings and conduct by means of speech.

2. Test the following definitions :—

(*a*) A net is a reticulated fabric decussated at regular intervals.

(*b*) Porosity is the property which bodies possess of having pores.

(*c*) A gentleman is a person who moves in good society.

(*d*) Mercury is not a solid metal.

(*e*) Humour is thinking in jest, while feeling in earnest.

All these definitions are fallacious for the following reasons :—

(*a*) The definition is obscure. Moreover, it moves in a circle, since net and reticulated are synonymous terms.

(*b*) It is a circle in definition.

(*c*) It is an accidental definition or description.

(*d*) It is a negative definition.

(*e*) It is a description.

§ 8. **Exercises.**

1. Distinguish between Definition and Description. Indicate the Formal and Material Conditions of Definition.

2. Define the following terms :—

Animal, Plant, Rock, House, Table, Punishment, Book, Figure, Food, Society, University, Square, Friend, Child.

3. Test the following definitions :—

(1) Words are the signs of thought.

(2) A gentleman is a man living in a decent style.

(3) A gentleman is a man who wears English clothes.

(4) Opposed propositions are those which differ in quantity and quality.

(5) A judge is a lawyer who exercises judicial functions.

(6) Civilization consists in eating with a knife and a fork.

(7) Black is the opposite of white.

(8) A triangle is a figure having three equal sides.

(9) Life is a mode of activity.

(10) Pleasure is the absence of pain.

(11) Oxygen is a gas.

(12) Logic is the science of thought.

(13) Gold is a precious metal.

(14) A disjunctive syllogism is a syllogism whose major premise is a disjunctive proposition.

(15) A candle is a kind of light used before gas was invented.

(16) A circle is a plane figure bounded by one line.

(17) Conversion is the changing of terms in a proposition.

(18) A citizen is a man who pays taxes.

(19) Death is the extinction of vital forces.

(20) A soldier is a man brave and ready to die for his country.

(21) A dog is an animal of the canine species.

(22) Life is the opposite of death.

(23) A dog is a domestic animal that barks.

(24) Virtue is acting rightly.

(25) Ignorance is a blind guide.

(26) Life is the sum of the vital functions.

(27) Humour is the perception of unexpected incongruities.

(28) Induction is the colligation of facts by means of an appropriate conception.

(29) Life is bottled sunshine.

(30) Logic is the science of proof or evidence.

(31) Logic is the science of argument, *i.e.*, of inference and proof.

(32) Man is a rational biped.

(33) Necessity is the mother of invention.

(34) A net is a collection of holes strung together.

(35) Noon is the time when the shadows of bodies are shortest.

(36) Peace is the absence of war.

(37) Politeness is the oil that lubricates the wheels of society.

(38) The sun is the centre of the solar system.

(39) Gravity is a universal property of matter in virtue of which every body gravitates to every other.

(40) Man is an animal that makes clothes for himself.

(41) Rice is an article which is used as food in India.

(42) A periphrasis is a circumlocutory cycle of oratorical sonorosity circumscribing an atom of ideality lost in verbal profundity.

(43) Music is an expensive noise.

(44) Failure is but the want of success.

(45) A student is a youth attending an educational institution with books.

(46) The Controller of Examinations controls the University examinations.

CHAPTER XXVI.

DIVISION AND CLASSIFICATION.

§ 1. **Definition, Division, and Classification.** If Definition is concerned with the connotation, Classification and Division are concerned with the denotation, of a term. In the one case, our aim is to explain the proper meaning of a term by indicating the common and essential attributes of the class denoted by it; while, in the other, our aim is to connect an individual (or class) with a class having affinity with it, so as to bring out in the easiest manner its points of similarity and difference. And, as the connotation and denotation of a term are closely connected with each other, Definition, Division, and Classification are also intimately related logical processes. In defining terms, we are aided by division and classification, as these bring before our mind the members having striking points of similarity and difference, which enter into the definition as genus and differentia. Again, definition in its turn renders a division and classification precise and accurate by laying down the characters on which sound division and classification should always be based. If classification is not a meaningless process, it involves a reference to definition, explicit or implicit, popular or scientific; and, in order that definition may be possible, we must be able to think together the members of the class

Definition unfolds the connotation, while Division and Classification systematize the denotation of a term.

Definition, Division and Classification are closely connected processes, which help one another.

which we want to define. It may be mentioned in this connection that popular classification, like popular definition, is often superficial and vague, while the scientific form in each case is comparatively sound and precise.

Division and Classification indicate the downward and the upward course in arranging things: in the one we proceed from higher to lower classes; while in the other, from individuals to classes.

Division and Classification, as indicated above, have both to do with a systematic arrangement of the denotation of a term. They are essentially the same process, only beginning at opposite ends. In Division we proceed from the higher to the lower class, until we come to *infima species* or the lowest groups; while in Classification, we proceed from individuals to classes, and from lower classes to higher classes, until we come to the *summum genus* or the highest class. They are both guided by a sense of what is necessary to constitute a class, *i.e.*, by an apprehension of the qualities which may be supposed as forming a sort of implicit, and often inaccurate, definition. Classification and Division may, accordingly, be viewed as representing in a condensed and convenient form the results of Definition. And we have already seen how Definitions may be regarded as embodying the conclusions of Inferences, inductive and deductive. Thus, Classification and Division may be taken as containing in a nutshell the information gathered by prior logical processes. (*Vide* Chap. XXV, § 1.)

Classification and Division embody the results of Definition and Inference.

§ 2. Generalisation, Induction, Explanation, Definition, and Classification.

In generalisation we predicate of a class what is found to be

Generalisation consists in proceeding from one or more instances to all members of the class, as

when on observing that fire burns or iron rusts in some cases, we conclude that it is equally true in all other cases. Such inferences, therefore, always rest on the detection of a 'common nature' in all these instances, by reason of which we are led to think that what is true of some is true of all. If we hastily suspect the presence of such a common nature, our generalization becomes precarious ; if, on the other hand, we determine such a nature by a careful examination of materials and a cautious employment of the inductive canons, our generalization turns out to be comparatively certain. Again, when the grounds of a generalization are not discovered, it may be accepted simply as an empirical truth not contradicted by experience ; but it cannot then be regarded as an induction proper. Induction requires that a generalization must stand the test of the Inductive Canons and so reveal a uniform causal connection among the factors or qualities constituting the 'common nature.'

true of some of its members.

A generalization rigorously proved by the inductive canons is viewed as Inductive.

Generalization in every form is closely connected with Explanation and Classification, since all of them aim, as we have seen, at the detection of the points of similarity. And, according as these points are fundamental or superficial, we characterise these processes as scientific or popular. Classification regards the points of similarity as fixed qualities, while Explanation treats them as in a process of genesis. In fact, the distinction between *qualities*, on which Classification is based, and *processes* or *activities*, on which Explanation is

Generalization, Explanation, and Classification are interconnected.

Classification contemplates the statical

aspect of things, while Explanation, their dynamical aspect.

based, is entirely relative: qualities refer only to what we attribute to an object by reference to the effects on our senses (*e.g.*, weight, colour), while activities refer to some power or energy by reason of which these effects are believed to be produced (*e.g.*, gravity, light). Now, in the case of Explanation, we refer to the causes which bring about an event or produce an object; while in the case of Classification, we refer to the features as they are found in it. Thus, if Explanation be described as analysing Nature in its dynamic aspect, Classification may be described as analysing it in its static aspect. Both of them, properly carried out, bring the mind in harmony with Nature and both satisfy our curiosity and rationalise our memory.

Classification, Definition and Explanation influence one another.

We have already seen that Classification and Definition are closely connected. In the one we refer an individual or a group to a class supposed to possess fixed characters or fundamental attributes, while in the other we explicitly state these so as to convey a clear and accurate idea of the class as distinguished from other classes. And both these functions subserve the end of Explanation and are exercised by means of correct generalization and induction. Classification as well as Definition has, accordingly, been described at times as but a compendious form of Explanation. As means of communicating knowledge, Explanation is rather lengthy, Definition comparatively brief, while Classification is condensed to the utmost limit; what is expressed in the case of an Explanation in several sentences is expressed in a Defi-

Classification and Definition are compendious forms of Explanation.

nition usually in one sentence, while in a Classification the sense is conveyed merely by a term or name.

§ 3. Character and Forms of Classification. Classification is the systematic arrangement of objects in groups according to certain points of similarity and difference. Every classification is relative to some end in view. A painter, for example, may classify birds into animals that are beautiful and those that are not; while a musician may classify them into singing creatures and those that are not so. Man, for example, may be classed under vertebrate animals by the zoologist, while under moral beings by the moralist. Popular classification is mainly determined by the special character of experience and interestingness. Popular classification is thus to a great extent capricious, superficial, and variable.

Classification is systematic arrangement of objects in groups according to certain points of similarity and difference.

Popular classification is superficial and variable.

(1) Classification may be either (*a*) general or (*b*) special. (*a*) **General** or **scientific classification** aims at knowledge: objects are classified according to their important and prominent qualities. The **golden rule of scientific classification** is that objects should be classified according to their most important and most numerous points of community. (*b*) **Special classification** has reference to some definite or particular end in view, as when horses are classified into animals that are fast and those that are slow.

General or Scientific classification is based on many important points of similarity. Golden rule of classification.

Special classification is relative to some particular end.

(2) Classification has otherwise been distinguished into (*a*) natural and (*b*) artificial. (*a*) **Natural classification** is based on real points of similarity

Natural classification is based on numerous and impor-

tant points of similarity, characterizing a group as fixed by Nature.

and difference, *i. e.*, on numerous and important points of community among objects which differ from others. The Natural Kinds (Minerals, Plants, and Animals), for example, constitute distinct classes, in as much as they are separated from one another by prominent and numerous features. According to Mill, the qualities of Natural Kinds are really inexhaustible. Natural classification is allied to the scientific form referred to above. (*b*) **Artificial classification,** on the other hand, is based on some feature arbitrarily selected as the principle of classification. As when balls are classified into things that are red and those that are not so. Artificial classification is thus allied to the special form indicated above.

Artificial Classification is based on some feature or features relative to a definite purpose.

Natural Classification is supposed to be based on the natural constitution of things: while Artificial, on arbitrary human requirements.

The distinction between Natural and Artificial Classification is based on the assumption that the natural classes are more or less fixed, being characterized by a definite group of qualities as given by Nature, while the artificial classes are formed by pure caprice or the requirements of man. In the one, we adapt our classification to what is settled by Nature; in the other, we arbitrarily readjust the natural groups to suit our own ends. The origin of this distinction may be traced to Mill's doctrine of *Real* or *Natural Kinds*, indicating classes (such as Minerals, Plants, and Animals) on which their special features have been permanently stamped by Nature. And the peculiarity of Natural Kinds is that they are characterized by an indefinite number of common attributes, while artificial classes have at most only a few points of simi-

The distinction may be traced to Mill's view of Natural Kinds.

larity. Thus, if the qualities of man, plant, metal, or sulphur are numerous, those of such classes as round things, tables, libraries, or pens are comparatively few.

It may be mentioned, however, in this connection that the distinction between Natural and Artificial Classification is not quite sound. (*a*) Even if we assume that Nature has unalterably fixed the distinguishing qualities of the Natural Kinds, still in classifying them we have to select only some qualities as the ground of our classification, ignoring the rest. And, in the case of artificial classification, we have also to select from among the possibilities furnished by Nature. In classifying things into, say, large and small, we have to take into account the size in any case, which is supplied by Nature. (*b*) The view—that in natural classification the points of similarity are numerous, while in artificial classification they are few—is not strictly true. If we closely observe, we find that the points of difference between such artificial classes as rupees and sovereigns, Englishmen and Santals, Colleges and *Pathshalas* are not few, each group being characterized by many qualities. (*c*) We find, however, as a matter of fact, that the different natural classes which we treat as separate, have often important points of similarity; and the modern Theory of Evolution tries to show that the different grades of being are developments out of one primitive stock.

The distinction, however, is not strictly correct, as all classification is due to human invention or choice.

We find, accordingly, **Classification by Series** emphasized now-a-days. As the entire animal life, for example, represents one kind of

Classification by Series is the arrangement of classes in a

graduated scale according to the variation of some feature or features.

being manifested in different degrees or grades, the arrangement of its different forms in a series, according to the more or less perfect manifestation of its fundamental qualities, is more conducive to the discovery of its essential attributes than a mere detached enumeration of the several forms. Thus, the vertebrate animals may be arranged in a serial order beginning with the highest class, mammals, and then proceeding to birds, reptiles, amphibia (*e.g.*, frogs), and fishes. Such an arrangement enables us to discover easily, by the Method of Concomitant Variations, the fundamental qualities which characterize the kind of being manifested in different forms. We discover, for example, in this way that the essential qualities of the vertebrata are the possession of a backbone, a nervous system, jaws as parts of the head, and four limbs disposed in pairs. "The requisites of a classification intended to facilitate the study of a particular phenomenon," says Mill, "are, first, to bring into one class all Kinds of things which exhibit that phenomenon, in whatever variety of forms or degrees; and secondly, to arrange those Kinds in a series according to the degree in which they exhibit it, beginning with those which exhibit most of it, and terminating with those which exhibit least." (*Logic*, II, p. 289.) The Evolutionist carries Classification by Series to its farthest limit when he tries to weave a connected account of the entire universe. (*Vide* Chap. XVIII, § 7.)

It is favourable to the discovery of the fundamental qualities of a Class or Order.

Deductive Classification or Division

(3) Classification may again be divided into (*a*) Deductive and (*b*) Inductive. (*a*) **Deductive Classi-**

fication, or what is ordinarily called **Division**, consists in dividing a class into sub-classes, and these sub-classes again into smaller groups, and so on, until we come to the lowest species. (*b*) **Inductive Classification**, or what is sometimes simply called **Classification**, consists in arranging individual objects into groups or classes according to their points of similarity and difference. These groups are again brought together under higher groups, and so on, by reference to the prominent points of similarity. If deductive classification proceeds from the more general to the less general, inductive classification proceeds from the less to the more: if the one be described as a downward process, the other may be described as an upward one.

is the breaking up of a class into sub-classes.

Inductive Classification (or what is simply called Classification) is the arrangement of individuals in groups according to certain points of affinity.

The distinction between Deductive and Inductive Classification, like that between Deduction and Induction, is not an absolute one. Deductive Classification involves the inductive form, and Inductive Classification involves the deductive type. When, for example, we divide animals into rational and irrational, we assume the presence or absence of rationality as the principle of Classification, which must have been gathered by Induction. Inductive Classification, similarly, implies the deductive form. When, for example, we classify tables, chairs, and benches as furniture, we assume that there is a bond of similarity among them, by reason of which the smaller classes follow from the larger one. There is a hypothesis as to the ground of resemblance under which the things

The distinction between Deductive and Inductive Classification is relative and not absolute.

to be classified may be brought, *i.e.*, from which they may, so to speak, be deduced. Though Deductive and Inductive Classification are thus closely connected, yet we may call a classification Deductive or Inductive according as it prominently illustrates this or that method.

Whewell maintains that Classification proceeds by type, which pre-eminently represents the Characteristics of a class.

§ 4. Is Classification by Type or by Definition? It is a matter of controversy among logicians whether we classify by reference to type or definition. Whewell contends that Classification proceeds by type. A Type* is an eminent example of a class embodying its characteristics in a conspicuous and complete form. In order to define food, for example, we may take milk, rice or bread as a type, since it embodies the qualities of food as nourishment in a prominent form, and we may then try to bring other objects under the class

* "The type of each genus," says Waterhouse, "should be that species in which the characters of its group are best exhibited and most evenly balanced." Mill also writes, "We must consider as the type of the class, that among the Kinds included in it, which exhibits the properties constitutive of the class, in the highest degree; conceiving the other varieties as instances of degeneracy, as it were, from that type; deviations from it by inferior intensity of the characteristic property or properties. For every phenomenon is best studied (*cæteris paribus*) where it exists in the greatest intensity. It is there that the effects which either depend on it, or depend on the same causes with it, will also exist in the greatest degree. It is there consequently, and only there, that those effects of it, or joint effects with it, can become fully known to us; so that we may learn to recognise their smaller degrees or even their mere rudiments, in cases in which the direct study would have been difficult or even impossible. Not to mention that the phenomenon in its higher degrees may be attended by effects or collateral circumstances which in its smaller degrees do not occur at all, requiring for their production in any sensible amount a greater degree of intensity of the cause than is there met with. In man, for example, (the species in which both the phenomenon of animal and that of organic life exist in the highest degree) many subordinate phenomena develop themselves in the course of his animated existence, which the inferior varieties of animals

(food) by reference to it (*i. e.*, milk, rice or bread). So in classifying flowers, we may take the rose or the lily as the type; or, in forming the class 'felidæ', we may take the tiger as the type and then bring the lion, the leopard, the cat, the puma under the class as they more or less resemble the type. Likewise, in classifying things into solid, liquid and gaseous, we select their types, such as stone, water and hydrogen respectively; and things resembling one or other of these types are classified as solid, liquid or gaseous. A class is thus formed by the type round which individuals resembling it are brought together. "Natural Groups," says Whewell, "are best described, not by any Definition which marks their boundaries but by a Type which marks their centre. The Type of any natural group is an example which possesses in a marked degree all the leading characters of the class. A Natural Group is steadily fixed, though not precisely limited; it is given in position, though not circumscribed; it is determined, not by a boundary without, but by a central point within;—not by what it strictly excludes, but by what it

do not show. The knowledge of these properties may nevertheless be of great avail towards the discovery of the conditions and laws of the general phenomenon of life, which is common to man with those inferior animals. And they are, even, rightly considered as properties of animated nature itself; because they may evidently be affiliated to the general laws of animated nature; because we may fairly presume that some rudiments of feeble degrees of those properties would be recognised in all animals by more perfect organs, or even by more perfect instruments, than ours; and because those may be correctly termed properties of a class, which a thing exhibits exactly in proportion as it belongs to the class, that is, in proportion as it possesses the main attributes constitutive of the class." (*Logic*, II, pp. 291-292.)

eminently includes ;—by a Type, not by a Definition." (*Novum Organon Renovatum*, pp. 21—22.)

Mill holds that Classification is based on definition, indicating the essential qualities by reference to which members are to be grouped together.

Mill, however, contends that Classification is based on definition. The essential attributes or characteristics of a species must first be definitely laid down, and then objects should be classified by reference to them. Mill's method presents the difficulty of classing individuals which prominently resemble a class but may not possess all the attributes included in the Definition. Should we, for example, regard idiots and lunatics as men? Is sponge a plant or an animal? Mill, no doubt, would question the propriety of including such individuals in the class : scientifically they should be left out. But he admits that classification is often suggested by type, though it is to be corrected by definition.

Whewell's view represents the ordinary, while Mill's the logical, form of classification.

It appears that Whewell refers to the ordinary method of classification. Ordinarily, we never take the trouble of carefully examining the individual objects and gathering their connotation before classifying them. We classify by reference to the average impression or type. The earlier classifications, in the history of the race as well as of the individual, are of this form. Mill's account of classification, on the other hand, refers to the correct or ideal form of classification. Classification should always be by reference to the deep-seated and numerous points of community embodied in a definition. If Whewell's account indicates the actual form of classification, Mill's indicates the logical form. With regard to classification by

types, Jevons writes, "Perplexed by the difficulties arising in natural history from the discovery of intermediate forms, naturalists have resorted to what they call classification by types. Instead of forming one distinct class defined by the invariable possession of certain assigned properties, and rigidly including or excluding objects according as they do or do not possess all these properties, naturalists select a typical specimen, and they group around it all other specimens which resemble this type more than any other selected type...It would be a great mistake to suppose that this classification by types is a logically distinct method. It is either not a real method of classification at all, or it is merely an abbreviated mode of representing a complicated system of arrangement. A class must be defined by the invariable presence of certain common properties. If, then, we include an individual in which one of these properties does not appear, we either fall into logical contradiction, or else we form a new class with a new definition. Even a single exception constitutes a new class by itself, and by calling it an exception we merely imply that this new class closely resembles that from which it diverges in one or two points only." (*Principles of Science*, pp. 722-723.)

Jevons agrees with Mill.

§ 5. **Classification Modified by Evolution.** The old doctrine of classification should be modified to suit the modern Theory of Evolution. Classes can no longer be viewed as essentially and fundamentally distinct, possessing different or unlike attributes. The different

The older method of classifying the different species as radically distinct, characterized by essentially

different qualities, has been modified by the Theory of Evolution, which classifies them as members of the same family by reference to their 'affinity' or proximity of descent.

species are not radically distinct; they are but different developments out of the same stock, under varying conditions. Consistently with the theory of evolution, when we classify, say, plants or animals, by reference to 'affinity,' 'affinity' should be understood as implying nearness of descent from the same stock. The animal kingdom, for example, may be regarded as a family tree illustrating a long line of ancestors; and we should classify the different kinds of animals into higher or lower classes according to the proximity or remoteness of their descent from the primitive stock. Men may thus be brought under apes as their immediate ancestors; and apes may be brought under quadrumana, that may similarly be brought under mammals, and so on. In fact, the character of classification is altered: it is no longer based on empirical laws having reference to the fixed and unalterable characters of the different classes; it rests rather on derivative laws following from causation. If previously classification was essentially inductive and empirical; as modified by evolution, classification becomes rather deductive and derivative.

Classification thus becomes deductive and derivative, while previously it was inductive and empirical.

§ 6. Classification, Conception, Abstraction, and Generalization. Classification and Conception are evidently very closely connected with each other. We classify objects when we bring them under general notions or concepts. Without a conception there can never be a classification. In order, however, to form a concept and thus to classify things, we must have recourse to abstrac-

Classification and Conception are closely connected, since to classify things is to connect them by a suitable conception.

Conception

tion and analysis. (*Vide* Chap. IV, § 3, Chap. XVI, § 4, and Chap. XXVIII, § 2.)

involves abstraction and analysis.

Abstraction should be distinguished from Analysis by the fact that, in it, we do not exhaustively consider all the constituent qualities, but we consider only some, withdrawing our attention from the rest. (*Vide* Chap. XVI, § 4.) Analysis thus involves abstraction at every step. There are differences of opinion with regard to the character of abstraction itself :—(*a*) some hold that abstraction means the *withdrawal of attention from* certain features ; (*b*) while others contend that abstraction implies the *direction of attention to* certain features. In the midst of this apparent diversity of opinion we find that there is a common point. The concrete exercise of attention in every case involves both a positive and a negative factor. We can, for example, never direct our attention *to* the stature alone of a person without for the time being withdrawing our attention *from* the other features, such as colour, form, &c. But if we still persist in raising the question, Which of these two aspects of attention—negative and positive—is indicated by abstraction ? then we may find a reply in the etymology of the term itself. The etymology of Abstraction (Lat. *abs* from, and *traho*, to draw) suggests that we are to understand by it the negative aspect rather than the positive.

Different interpretations of Abstraction

which has really a positive and a negative aspect.

A question has been raised as to the relation of Abstraction to Generalization. Without entering into the perplexing psychological aspect of the relative priority of either of these two processes

(which is outside the scope of our study), we may simply observe that these two processes are very intimately related to each other. Whenever we withdraw our attention from certain features, we evidently have general ideas of them, without which we can never at all be aware of the features from which to withdraw our attention. Again, it is by withdrawing our attention from the individual peculiarities that we direct our attention to the common features and thus form general ideas of classes. Abstraction and Generalization thus go hand in hand. "What we understand by Generalization," observes Dr. Venn, "is to be interpreted in a wide sense: we must be careful to associate with it all that process of analysis, and of the requisite exclusions, by which alone generalization can be made trustworthy." (*Empirical Logic*, p. 356.)

Abstraction and generalization are inter-connected.

§ 7. The Rules or Conditions of Classification. The rules of scientific classification are the following :—

(1) *Place together in groups those things that possess in common the most numerous and the most important qualities.* This is the golden rule of definition and classification alike. To classify objects on the ground of some one quality, such as the presence of four legs or warm blood, is not so useful as to bring together things having several important points of community (*e. g.*, Mammalia, Felidæ, Rosaceæ). And to classify objects merely by reference to their superficial or outward features (such as colour, form, hair), however serviceable it may be for practical purposes, is scientifically useless.

(1) The golden rule of classification is to group together things having the most numerous and the most important points of similarity.

(2) *Connect the groups having close affinity and separate those that are marked by important points of difference.* This rule enables us to distinguish aright the classes which are characterized by prominent differences, though they may have also points of similarity by reason of which they may be viewed as subdivisions of a higher class. Thus, Birds are distinguished from Reptiles, though both of them are brought under the higher class, Sauropsida, 'characterized by the absence of gills, by having the skull joined to the vertebral column by a single occipital condyle, the lower jaw composed of several pieces, and united to the skull by means of a special (quadrate) bone and by possessing uncleated red blood corpuscles.'

(2) Distinguish groups by important points of similarity and difference.

(3) *Graduate the classification upwards, tracing smaller groups to higher or more general classes and continuing the process until a comprehensive class or kingdom is reached which shows the affinity.* Such a plan has the advantage of exhibiting in a comprehensive scheme the mutual relation of the subdivisions according to their fundamental points of similarity and difference.

(3) Graduate the classification upwards.

(4) *The proximity of subdivisions in a general scheme should be determined by the degree of their affinity or similarity; and their distance, by their difference or variation.* This rule implies that, in arranging the classes, we should place them so in a scheme of classification that their proximity should indicate their similarity, and their distance, their difference; and the degree of proximity or distance should indicate the degree of similarity or

(4) The proximity or distance of classes in a general scheme of classification should be determined by the degrees of their similarity and difference.

difference. Thus, instead of classifying triangles into equilateral, scalene, and isosceles, it would be better to classify them into equilateral, isosceles and scalene, showing the gradual descent in the equality of sides. A sliding scale is better than abrupt transitions. The truth of these remarks will appear from the following table of the classification of Animals given by Mr. Read in his *Logic* :—

KINGDOM : Animal Kingdom

SUB-KINGDOM : Vertebrates — Invertebrates (5 Sub-kingdoms)

Sauropsida — Ichthyopsida

CLASS : Mammals — Birds — Reptiles — Amphibia — Fishes

SUB-CLASS : Placental — Implacental

DIVISION : Monodelphia — Didelphia — Ornithodelphia

ORDER : Quadrumana — Rodentia — Carnivora — Ungulata — Cætacea, *etc.*

SECTION : (Family) Pinnigrada (Seals, *etc.*) — Plantigrada (Bears, *etc.*) — Digitigrada

GENUS : Mustelidæ (Weasels, *etc.*) — Viverridæ (Civets, *etc.*) — Hyænidæ — Canidæ — Felidæ

SPECIES : Lion — Tiger — Leopard — Puma — Lynx — Cat, *etc.*

VARIETY : African — Syrian — Cave-lion (extinct).

It may be mentioned here that the term 'species' is used in Division or Deductive Classification to express any class which may be viewed as included in another, the terms species and genus being entirely relative. (*Vide* Chap. IV, § 5.) In (Inductive) Classification, 'species' usually stands for the lowest class alone, the highest being called kingdom. If, in any case, the lowest class is still divisible into smaller classes, these are named as sub-species or varieties. The names of the different classes in Inductive classification may be arranged in order of generality thus—(1) Kingdom, (2) Sub-kingdom, (3) Class, (4) Sub-class, (5) Division, (6) Order, (7) Section or Family, (8) Genus, (9) Species, (10) Variety.

The term 'species' is used in Division to indicate any smaller class; but, in Classification, it represents the lowest class in a series.

§ 8 The Rules or Conditions of Division. Before proceeding to consider the rules of Logical Division, let us distinguish it from the other forms of Division with which it is liable to be confounded. Division is said to be either Logical, Physical, or Metaphysical. **Logical division** is the division of a class; it is the systematic arrangement of the denotation of a general term into groups or smaller classes according to the presence, absence, or varying degree of some important attribute. We may divide, for example, the class 'man' logically into the sub-divisions learned and other than learned, according to the principle—presence or absence of learning. Similarly, we may classify 'man' differently according to some other principle (*e.g.*, height or stature) and get the sub-divisions—tall, short, and of medium

Logical, Physical, and Metaphysical Division distinguished.

Logical Division is the systematic arrangement of the individuals of a class into subordinate groups on some fundamental principle.

Physical Division is the separation of an individual into its component parts.

size. While logical division is thus the division of a class, **physical division** or **partition** is the division of an individual into its constituent parts. Thus, physical division is always the division of an object indicated by a singular term. If, for example, we divide 'man' into the head, the trunk, and the limbs, we get an example of physical division. If physical division is the breaking up of an object into its component parts, **metaphysical division** or **analysis** is the enumeration of the constituent qualities of an object. Metaphysical division is illustrated when we divide 'man' into animality and rationality. It is but the enumeration of the attributes or qualities found in man. Similarly, 'chalks' may be logically divided into, say, drawing chalks and other than drawing chalks. The physical division of 'chalk' would be to break it up into its component parts, while its metaphysical division would be its analysis into its constituent qualities, such as whiteness, opacity, brittleness, &c.

Metaphysical Division is the analysis of an object into its constituent qualities.

As, in Logic, we are not concerned with physical partition or metaphysical analysis, we shall limit our inquiry to the determination of the conditions of logical division. The rules of logical division are :—

Rules of Logical Division :

(1) The term to be divided must be general.

The fallacy due to its violation is one of physical or metaphysical division.

(1) *The term to be divided must be general* : a class can be divided into sub-classes. The violation of this rule gives rise, as shown above, to the fallacy of *physical or metaphysical division*. It is apparent from this that a collective term, as such, cannot be logically divided. If we divide 'a library' into the books of reference, science, history, and fiction, we are guilty of physical division, as we

divide an individual into its parts. The division of 'the British Parliament' into the House of Lords and the House of Commons, or of 'a forest' into, say, the mango trees, pine trees, teak trees, and *sal* trees similarly indicates physical division.

(2) There should be a single principle of division.

(2) *There should be a single principle of division—a fundamentum divisionis.* Whenever we divide a term, we must accept as the principle of division a single line of thought—the presence, absence, or varying degree of some important attribute. When, for example, we divide 'men' into tall, learned, and virtuous, we transgress this rule, since we adopt more than one principle as the principle of division. The fallacy arising from the violation of this rule is technically called *the fallacy of cross division.* Similarly, the division of 'tables' into circular, wooden, marble, brown, and heavy, or of 'coins' into English, French, gold, round, and valuable illustrates the same fallacy.

The fallacy due to its violation is known as cross division

(3) The term divided must be predicable of each of the subdivisions.

(3) *The name of the class divided must be applicable to each of the sub-divisions.* When, for example, we divide 'men' into learned and other than learned, we can predicate the term 'man' of each of the sub-divisions. The violation of this rule gives rise to the fallacy of *physical or metaphysical division.* When, for example, we divide 'a table' into its top, legs, and framework, or 'gold' into its malleability, weight, and yellow colour, we cannot predicate the name 'table' of the top, legs, or framework, nor 'gold', of malleability, weight, or yellow colour.

Violation of this rule implies physical or metaphysical division.

(4) Division must be exhaustive.

(4) *All the sub-divisions taken together must make up the class divided*; otherwise the division is said to

The fallacy due to its violation is known as incomplete division.

be incomplete. If, for example, we divide 'men' into tall and short, the division may be said to be *incomplete*, in as much as men of medium size are left out of account. Similarly, to divide 'triangles' into equilateral and scalene, or 'the articles of food' into sweet and sour would be to commit such a fallacy, as the isosceles triangles, in the one case, and the things having some other taste, in the other, are passed over.

(5) The subdivisions must be mutually exclusive.

(5) *The sub-divisions must exclude one another*; otherwise we get the fallacy of what is called *overlapping division*. If, for example, we divide men into tall, honest, and industrious, we are guilty of this fallacy, in as much as industrious men may be tall or honest, honest men may be tall or industrious, and tall men may likewise be honest or industrious. Thus, the sub-classes overlap, *i.e.*, they are not mutually exclusive. It may be mentioned in this connection that the transgression of this rule involves also the violation of the second rule given above, which may be regarded as the principal rule of Division.

The fallacy due to its violation is known as overlapping division.

(6) A class should be subdivided into proximate groups.

(6) *A class should be divided into its proximate sub-classes*, as otherwise a division is not likely to serve any useful purpose. The transgression of this rule gives rise to the error known as *division by a leap*. If, for example, we divide 'animals' into learned men and those that are not learned, the division is practically useless. We may divide 'animals' into rational and irrational, or 'men' into learned and other than learned; but to divide 'animals' into those that are learned and those that are not so is, to say the least, fanciful and eccentric.

The fallacy due to its violation is known as division by a leap.

Whenever we logically divide a term, it is often difficult to ascertain, without a knowledge of the subject-matter of the term divided, whether the rules given above are satisfied or not. Hence a form of division known as division by dichotomy is suggested which, from its mere form, shows that the logical rules are satisfied. **Division by Dichotomy** (Gr. *dicha*, in two, and *temno*, to cut) is the division of a class into two contradictory sub-classes, as when we divide men into tall and not-tall, tables into rectangular and not-rectangular, horses into fast and not-fast, days into bright and not-bright.

Division by Dichotomy is the division of a class into contradictory sub-classes.

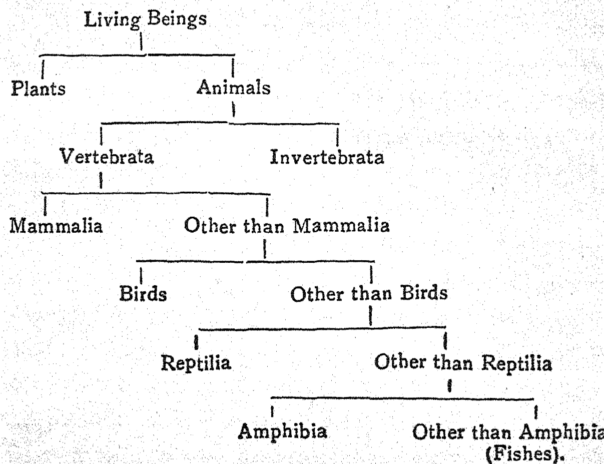
As contradictory terms are mutually exclusive, and as, taken together, they make up the whole universe, we are sure, in the case of division by Dichotomy, that the several logical rules are observed. It is based on the Principle of Excluded Middle. (*Vide* Chap. II, § 6.)

It ensures formal correctness.

If the table in the preceding section illustrates how the different forms of animal life may be systematically grouped under different classes, the following tables illustrate how a higher class may be systematically subdivided into smaller groups by the process of Dichotomy :—

Illustrations.

- Figure
  - Rectilineal
    - Triangular
      - Equilateral.
      - Not-equilateral.
    - Not-triangular.
  - Not-rectilineal
    - Circular.
    - Not-circular
      - Oval.
      - Not-oval.



Uses of Classification:

(1) It is a condensed and compendious form of explanation. It thus helps comprehension.

§ 9. Uses and Limits of Division and Classification.

The *uses of Classification* are briefly the following.—(1) Classification by indicating the important points of similarity and difference, enables us to understand things aright. Classification, as we have said, furnishes the necessary information in a compendious form: it is but an abbreviated explanation. Whenever we refer an object to its appropriate class, we know at once its important points of similarity and difference and thereby we understand it. The difference between Explanation and Classification lies (*a*) in the one being an explicit and fully expressed, while the other being an implicit and condensed, mode of accounting for things, and (*b*) in the one being concerned with phenomena or changes, while the other, with things as they are found or given. As Mr. Read says, "Explanation ana-

Points of difference between Explanation and Classification: (*a*) the one is explicit and fully expressed, while the other is implicit and condensed (*b*) the one

lyses Nature in its dynamic, Classification in its static aspect. In both cases we have a feeling of relief." (*Logic*, p. 303.)

has to do with processes, while the other with existing things.

(2) Classification is an important aid to memory. Whenever we refer an object to a class, we render its retention and subsequent revival easy. To remember the innumerable objects individually is difficult, if not impossible; but to remember them by reference to their classes or points of similarity is comparatively easy.

(2) Classification is an aid to memory.

(3) Classification gives control over the contents of memory. By marshalling things into classes we can readily find them out when wanted. A poet or a scientist, for example, may thus look in the right direction for a simile or an appropriate example. Classification suggests also hypotheses by analogy. It, accordingly, facilitates communication and explanation. This use is illustrated also in Division.

(3) It facilitates exposition and discovery.

Uses of Division:

The *uses of Division* are:—(1) Division gives us a definite knowledge of the denotation of a term and the differences which may exist among its several groups. It thus renders the application of a general term more precise or accurate. (2) Division is an important aid to the specialization of inquiry and thus to the discovery of secondary laws, which, as we have seen, are of great practical value. [*Vide* Chap. XXIII, § 5.] (3) It enables us also to think and reason systematically by reference to the distinct subdivisions of a class. It thus prevents confusion and is an aid to the clearness of thought.

(1) It renders the application of a term precise by giving us a definite knowledge of its denotation.

(2) It favours the specialization of inquiry and the discovery of secondary laws.

(3) It enables us to think and reason

clearly and correctly.

Division and Classification are always relative to certain ends.

It should be remembered in this connection that Division and Classification, when based on numerous and important points of similarity among the objects divided or classified, are more or less the work of the mind and are thus relative to definite interests, scientific or otherwise. 'Men,' for example, may be classified under bipeds, mammals, or intelligent beings according to the end in view; and 'men' may likewise be divided into Christians and non-Christians, civilized and uncivilized, or honest and dishonest. At times Division or Classification is scientific, being based on several essential points of similarity; and, at times, it may be practical, designed to serve some desirable end. Thus, birds may be classified by reference to their notes by a musician, their plumage by a painter, and their structure or habits by an ornithologist. We may similarly classify the subject-matter of a book in one way in the 'Contents' and in a different way in the 'Index.' It may be mentioned in this connection that an **Index Classification** has the advantage of limiting the possible classes to the number of the alphabet in a language (26 in English) and of arranging them in regular succession, so that they can be easily found out and their definite characters, promptly known. The Index Classification is ordinarily a secondary form, which is required to help the primary form of Classification based on the golden rule. Thus, the classification of plants or animals is primarily based on deep-seated and fundamental attributes discovered often by patient research involving dissection, micro-

Index Classification is often an important aid to the other forms of classification.

scopical examination, and careful comparison of different specimens. Similarly, the subject-matter of a book may be distributed in different chapters according to some fundamental principle determining the division of topics. But, to a beginner, the essential attributes of the different classes and the plan of division or classification would be a sealed book, if he be not aided by an *analytical key* which serves as an index or guide to the different divisions or classes and their fundamental qualities. We find, accordingly, the Index Classification adopted for the diagnosis of plants, animals, diseases, *etc.*, and hence it is known also as *Diagnostic Classification.* The different signs of structural differences or of morbid processes are recorded in the Index, which enables us to determine the character of a plant, animal, or disease by reference to the concurrence of the signs or symptoms. We thus infer the character of a plant, animal, or disease, by observing, say, the flowers and leaves, the teeth and limbs, the pulse, tongue, and temperature, in any case, and by comparing the characters observed with those recorded in the Index. (*Cf.* The Linnæan System of Botany.)

The Index Classification often enables us to discover the important qualities of a class by reference to their signs or symptoms. It is generally used in Botany, Zoology, Medicine, &c.

Having indicated the uses of Division and Classification, let us now consider their limits. The *limits to classification* are the limits to definite and precise knowledge. "A full classification," says Jevons, "constitutes a complete record of all our knowledge of the objects or events classified, and the limits of exact knowledge are identical with the limits of classification." [*Principles of Science,*

Limits of Classification

(1) Ignorance of definite characters.

(2) The marginal instances.

(3) Composite objects whose constituents are combined in varying proportions.

(4) Varying phenomena whose composition is not known.

p. 731.] (1) When we cannot satisfactorily ascertain the characters of an object or phenomenon, we cannot refer it to a particular class. Thus, there are difficulties in determining whether ether is material, or sponge, an animal. (2) The marginal instances also do not admit of easy classification. [*Vide* Chap. XXV, § 2.] (3) Composite objects whose constituents are combined in varying proportions cannot easily be referred to their classes. "Granite," for example, "is a mixture of quartz, felspar, and mica, but there are hardly two specimens in which the proportions of these three constituents are alike, and it would be impossible to lay down definitions of distinct species of granite without finding an infinite variety of intermediate species. The only true classification of granites, then, would be founded on the proportions of the constituents present and a chemical or microscopic analysis would be requisite, in order that we might assign a specimen to its true position in the series." [Jevons, *op. cit.*] (4) Still more difficult is it to classify a varying phenomenon whose composition is not known. "If we attempt to classify tastes, we may rudely group them according as they are sweet, bitter, saline, alkaline, acid, astringent, or fiery; but it is evident that these groups are bounded by no sharp lines of definition. Tastes of mixed or intermediate character may exist almost *ad infinitum*, and what is still more troublesome, the tastes clearly united within one class may differ more or less from each other, without our being able to arrange them in subordinate genera and species. The

same remarks may be made concerning the classification of odours, which may be roughly grouped according to the arrangement of Linnæus as, aromatic, fragrant, ambrosiac, alliaceous, fetid, virulent, nauseous. Within each of these vague classes, however, there would be infinite shades of variety, and each class would graduate into other classes. The odours which can be discriminated by an acute nose are infinite; every rock, stone, plant, or animal has some slight smell, and it is well known that dogs, or even blind men, can discriminate persons by slight distinctive odour which usually passes unnoticed." [*Ibid.*, p. 732.] (5) An elementary experience or the summum genus cannot be brought under a higher class. (*Vide* Chap. XXIV, § 4.) *The Limits of Division* are evidently (1) the infima species, which cannot be further subdivided into groups, (2) the ultimate experiences, and (3) composite things which are too peculiar to admit of subdivisions.

(5) An elementary experience or the summum genus.

Limits of Division: (1) Infima species, (2) ultimate experiences, and (3) composite things which are unique.

§ 10. **Hints for Working out Exercises.** (1) To determine in any case whether we are concerned with Classification or Division, we should observe whether the procedure is from individuals to classes, from lower classes to higher ones, or from higher classes to lower groups.

(2) To ascertain whether, in any case, there is logical, physical, or metaphysical division, we must examine whether the term divided is singular or general. The division of singular terms—whether concrete, abstract, or collective—implies physical or metaphysical division, while the division of a class indicated by a general term into subordinate groups implies logical division. A

division is physical or metaphysical according as the component parts of an individual or the constituent qualities of an object are enumerated.

(3) As a student cannot be expected to have always an adequate knowledge of the subject-matter of the term to be divided, it is advisable generally to divide terms by Dichotomy.

(4) Division must stop at the smallest groups (*infimæ species*); to proceed further would be enumeration and not division.

(5) We should never overlook the golden rule in the case of Classification and a *fundamentum divisionis* in the case of Division.

Illustrations.

I. (*a*) Divide and (*b*) classify (1) 'material bodies,' (2) 'men', (3) 'syllogisms,' (4) 'tables,' and (5) 'languages.'

1. (*a*) '*Material bodies*' may be divided thus:

- Material Bodies
  - Solids
  - Non-solids
    - Liquids
    - Non-liquids
      - Gases
      - Non-gases.

Good

(*b*) '*Material bodies*' may be classed under 'Bodies' or 'Substances.'

2. (*a*) '*Men*' may be divided thus:

- Men
  - Europeans
    - Englishmen
    - Non-Englishmen.
  - Non-Europeans

(*b*) '*Men*' may by classed under Bimana or two handed mammals.

3. (*a*) '*Syllogisms*' may be divided into Pure and Mixed. For subdivisions of these see Chap. XI, § 4 and Chap. XII, § 1.

(*b*) '*Syllogisms*' may be classed under Deductive Inferences.

4. (*a*) '*Tables*' may be divided thus:

- Tables
  - Those that are round
    - Those that are made of marble
    - Those that are not made of marble.
  - Those that are not round

(*b*) '*Tables*' may be classed under articles of furniture.

5. (*a*) '*Languages*' may be divided thus:

- Languages
  - Aryan
    - Teutonic
      - English
      - Non-English.
    - Non-Teutonic
  - Non-Aryan

(*b*) '*Languages*' may be classed under a system of signs.

II. Test the following divisions:—

(*a*) Plants into root, stem, and branches.

(*b*) Europeans into Englishmen, Frenchmen, and Germans.

(*c*) Men into Christians, Mahomedans, Hindus, Englishmen, and Dutch.

(*d*) Tables into round, marble, heavy, and black.

(*e*) Orange into round form, yellow colour, sweet taste, smooth surface, and peculiar smell.

(*a*) This is physical partition, breaking rule 1.

(*b*) This is incomplete division, violating rule 4.

(*c*) This involves the fallacies of cross division and incomplete division, violating rules 2 and 4.

(*d*) There are no less than four bases of divisions here—(1) form, (2) material, (3) weight, and (4) colour. It thus involves the fallacy of cross division. It is moreover incomplete.

(*e*) It is metaphysical analysis.

§ 11. Exercises.

1. Exhibit the nature and uses of Classification. Point out its relation to Definition.

2. Give a clear account of the principles of Scientific Classification. What is Classification by Series?

3. Give an account of Natural Classification, explaining what is meant by the 'essential' or 'fundamental' characters as the basis of Classification.

4. What is meant by a Natural Kind or Class? To what extent is the distinction between Natural and Artificial Class tenable?

5. Exhibit the procedure in Natural Classification and explain in this connection the distinction between Definition and Type.

6. Distinguish the province and aims of Classification from those of Division. How would you proceed when (*a*) dividing and (*b*) classifying the group 'animals'?

7. Determine the relation of Classification to Abstraction and Conception. Point out the bearing of Classification on Generalization.

8. How are Classification and Division related to Induction? Are they connected in any way with Definition?

In writing an essay you apply both Definition and Division : explain for what purposes you apply them.

9. Distinguish Logical, Physical, and Metaphysical Division. Explain the nature and uses of Logical Division.

10. Point out the principal errors incidental to Classification and Division. What is a Cross Division ?

11. What is Division by Dichotomy ? Why is this form of Division considered as specially suitable to Deduction ?

12. 'No such things as classes exist in nature ; every classification depends upon the purpose with which it is made.' Explain and discuss this statement.

13. What in your view is the real distinction between a Natural and an Artificial Classification ? How is Classification related to Explanation ?

14. Has Scientific Classification been modified in any way by the Theory of Evolution ? Illustrate your remarks by an example.

15. Distinguish between Division and Classification. Is Classification based on Type or on Definition ?

16. What do you understand by Index Classification ?

17. Divide and classify the following :—rectilineal figure, horse, book, virtue, house, term, succession, triangle, animal, river, tree, science, chair, pen, substance.

18. Test the following divisions :—

(1) India into Bombay, Madras, Bengal, and the United Provinces.

(2) Material Bodies into solids, liquids, and gases.

(3) Hindus into rich, poor, tall, and learned.

(4) Propositions into singular, universal, particular, affirmative, and necessary.

(5) Men into Aryans, Mongolians, Asiatics, and Christians.

(6) A stone into its colour, solidity, weight, and extension.

(7) Men into those who walk and those who crawl.

(8) Indian languages into Sanskrit, Mahratta, Tamil, Pali, Bengali, Uria and Hindi.

(9) Buildings into buildings of brick, buildings of stone public buildings, religious buildings, churches and law-courts.

(10) Sciences into physical, moral, and medical.

(11) Light into artificial light, sunlight, moonlight, gas-light, and electric light.

(12) The world into Asia, Africa and Europe.

(13) Books into entertaining and unentertaining.

(14) Men into those who lend and those who borrow.

(15) Religion into Christian, Mahomedan, Hindu, and Parsi.

(16) A person into bones, flesh, stomach, and head.

(17) Logic into Deduction, Induction, and Fallacies.

(18) Instruments into knives, scissors, spades, and shovels.

(19) Fruits into nutritious, sweet, fresh, and succulent.

(20) Metals into white, heavy, and precious.

(21) Triangles into equilateral and isosceles.

(22) Chair into seat, back, legs, and arms.

(23) Quadrilaterals into squares, rectangles, rhomboids, and parallelograms.

(24) Quinine into its bitterness, whiteness, and fineness.

(25) Figures into circles, triangles, quadrilaterals and pentagons.

CHAPTER XXVII.

TERMINOLOGY AND NOMENCLATURE.

§ 1. **Importance of Language.** We have seen that language is very closely connected with thought. (*Vide* Chap. I, § 1 and § 5.) The importance of language is illustrated in several ways :— (1) It is, as explained in Chapter I, necessary to the formation of thoughts. (2) It is also essential to their communication. We can never have an access into the minds of others but through their expression or speech. Thus, language—either natural or artificial—is always the necessary medium or vehicle for conveying the thoughts of one mind to another. (3) It is essential to the recollection of our previous experience. It is scarcely possible for us to remember things unless they are associated with language. Symbols are thus necessary not merely for the formation, but also for the retention and reproduction, of our ideas. (4) Language secures the precision of our thoughts. Our ideas are objectified, as it were, when they are clothed in language ; and definite expressions tend to render our thoughts definite and precise. (5) Language gives also facility to our thoughts. We can think more easily and quickly when we employ a system of symbols with but a sub-conscious reference to the ideas implied by them. As algebraic expressions facilitate calculation by the substitution of symbols for quantities which may be enormously large, so

Language and Thought are closely connected.

Language helps the (1) formation, (2) communication, (3) recollection, (4) precision, and (5) quick succession of thoughts.

words facilitate thinking by the substitution of symbols for ideas which may be of a very complex character. We should remember in this connection the distinction often drawn between **intuitive** and **symbolical thinking**. We think intuitively when we realize in thought what is described in words; but we think symbolically when we do not form ideas in our mind of what is expressed in language. In referring to a triangle or square, we may think intuitively; but, in referring to a chiliagon or a million pounds, we think symbolically. And often, in speaking and writing, we use expressions with but a vague reference to the corresponding ideas. The flow of ideas is thus materially aided by language.*

Distinction between intuitive and symbolical knowledge.

§ 2. **Definition, Classification, and Naming.** We have already seen that the different logical processes are all inter-connected. (*Vide* Chap. XXV, § 1.) Thus, Definition underlies Classification, and Classification prepares the way for Definition. And, Definition and Classification are possible only by means of symbols or names, which in their turn acquire a sense through Classification and Definition. When we classify objects,

Definition, Classification, and Naming are all inter-connected.

* We should remember that precision and facility seldom go together. Precision is secured more by intuitive thinking, while facility by symbolical. As symbolical knowledge gradually takes the place of the intuitive, the facility of thought increases, but more or less at the cost of precision, unless, by prior habitually correct use, the terms employed have acquired a fixed connotation. The validity of symbolical knowledge always depends on its possibility of being transformed into intuitive: if we can never realize in thought an account or description given in words, then it is to be presumed as incorrect. Mansel very appropriately likens symbolical knowledge to bank notes, whose worth always depends on their possibility of conversion into the current coins of a realm.

we attend to their points of similarity and difference, which constitute the meaning of the name applied to the class. Naming involves Classification, for to name is to refer an object or a group to a particular class. Classification again involves Naming. Comparison of individuals, their parts and qualities, is made with the help of general notions, which can be thought of only in connection with general names. Again, the product of classification is associated with a general name.

Connection between Naming and Classification.

The connection between Naming and Definition is also very close. A name always carries some sense in it; and this sense is clearly apprehended by means of a definition. If the meaning of a term is understood by reference to its definition, the definition too is conveniently retained in the compendious form of a name. Names may thus be viewed as the representative symbols of Definition and Classification. As a linguistic sign, a Name or Term enters into the composition of Propositions and Inferences and so may be regarded as simple or elementary; but, as representing a thought-product, embodying the results of prior logical processes, it is really very complex, suggesting all that we have already learnt.

Connection between Naming and Definition.

As a linguistic sign, a Term or Name seems to be simple; but, as standing for a product of thought, it is very complex.

§ 3. Terminology and Nomenclature.

The difference between Terminology and Nomenclature lies in the fact that the former refers to the terms used in describing the qualities or parts of things; while the latter, to the names of classes or individuals characterized by such qualities or possessing these parts. Plants or animals are

Terminology is a system of names for describing the qualities or parts of things, while Nomenclature is a system of names for the

classes or individuals themselves.

definitely known only by reference to their qualities (such as colour, form, size) or the composition or arrangement of their parts (such as the head, trunk, and limbs; the stalks, leaves, flowers, and fruits), a description of which falls within what is called Terminology. And, as popular names are often vague and indefinite, appropriate words are coined in many cases in the several sciences to express the qualities and parts aright, so that there may not be any difficulty in the due recognition of classes or individuals. We have thus such terms as *calyx*, *corolla*, *stamens*, *pistils*, *petals*, *sepals*, *perianth*, *pericarp* in Botany; *radius*, *ulna*, *sacrum*, *femur*, *fibula*, *tibia* in Human Anatomy; *vacuole*, *hydrosome*, *nectocalyx*, *polypites*, *ectoderm*, *endoderm*, *mesoderm* in Zoology. And there are also the special names of the different classes or groups, such as *Dicotyledons*, *Monocotyledons*, *Ranunculaceæ*, *Anonaceæ*, *Malvaceæ* in Botany; *Vertebrata*, *Mollusca*, *Annulosa*, *Infusoria*, *Protozoa* in Zoology; *Talc*, *Gypsum*, *Mica*, *Quartz*, *Topaz* in Mineralogy.

As popular names are generally vague, appropriate terms are coined in the different sciences for descriptive purposes.

Examples of scientific terms or names.

Though Nomenclature (from Latin *nomen*, name, and *calo*, to call) thus properly implies a system of names of individuals or classes, and Terminology (from Latin *terminus*, (here) term or appellation, and Gr. *logos*, discourse), a system of descriptive terms, indicating their qualities or parts, yet the two terms are at times used indefinitely to express the whole vocabulary of technical terms appropriated to any particular branch of art or science (*e.g.*, the nomenclature or terminology of painting or dyeing, of botany or chemistry).

'Nomenclature' and 'Terminology' are at times used indiscriminately for the entire system of technical terms used in a science or art.

§ 4. Popular and Scientific Use of Names. The popular use of terms is generally vague and uncertain, owing to the exigencies of ordinary life. This vagueness is due to two causes, one subjective and the other objective.

Popular use of terms is vague for two reasons :

(1) The subjective ground is to avoid the trouble of careful discrimination and also the trouble of increasing the stock of one's vocabulary. People are rather content with the use of such vague expressions as 'good,' 'nice,' 'awful' than to overburden their memory with precise expressions.

(1) Subjective which is to save proximate trouble ;

(2) Again, new and altered circumstances are often presented which require the use of common terms in slightly different or modified senses. Thus, 'pagan,' originally implying a villager, has gradually come to mean individuals not enlightened by the Christian faith (as villages are generally beyond the reach of such enlightenment) ; and 'salt,' though primarily referring to the familiar sea-salt, has gradually come to express the class of saline bodies in general.

and (2) objective, which is to meet the requirements of new circumstances.

We find, accordingly, that the signification of terms is altered either (1) by generalization or (2) by specialization. (1) In the case of *generalization*, part of the connotation of a term is gradually dropped, sometimes through ignorance and sometimes through thoughtlessness. An existing name, hitherto applicable to one class of objects, may thus be extended to another class partly resembling it. It is in this way that 'oil' has come to mean oils generally, instead of simply olive oil ; and 'parson,' clergymen at large, instead of merely the

Signification of Terms is altered or modified either (1) by generalization or (2) by specialization.

Examples of generalization.

rector of a parish. Similarly, 'Sybaritism,' 'Sandwich,' 'Drawcansir,' 'Silhouette', 'Mausoleum' have acquired a general signification, though originally they were connected with a particular group or individual. "The verb *transpire*," says Mill, "formerly conveyed very expressively its correct meaning, *viz.*, to *become known* through unnoticed channels—to exhale, as it were, into publicity through invisible pores, like a vapour or gas disengaging itself. But of late a practice has commenced of employing this word, for the sake of finery, as a mere synonym of *to happen*: "The events which have *transpired* in the Crimea," meaning the incidents of the war. This vile specimen of bad English is already seen in the despatches of noblemen and viceroys: and the time is apparently not far distant when nobody will understand the word if used in its proper sense." (*Logic*, II, p. 233.) (2) In the case of *specialization*, a fresh connotation is added to the general sense in which a term is primarily or ordinarily used. Thus, the word 'story' has come to mean a fictitious narrative from a mere narrative or account of an incident or event; and 'wit' has similarly come to signify 'facetiousness' from intellectual power in general. Cf. 'Pope'. We have seen that words generally are specialized when interpreted with reference to their context or the universe of discourse (*Vide* Chap. VII, § 1); and they acquire at times special senses with certain classes of men. "Thus, by cattle, a stage coachman will understand horses; beasts, in the language of agriculturists, stands for oxen; and birds, with some sportsmen,

for partridges only."* (Mill, *Logic*, II, p. 237.) Sometimes purely accidental circumstances invest terms with a peculiar connotation, as is illustrated in the case of 'cabal'. The reproach now associated with the term is due wholly to accident. Macaulay observes, "It so happened by a whimsical coincidence, that in 1671 the cabinet consisted of five persons, the initial letters of whose names made up the word *cabal*; Clifford, Arlington, Buckingham, Ashley, and Lauderdale. These ministers were, therefore, emphatically called the *cabal*; and it has never since their time been used except as a term of reproach." Thus, though the term (Fr. *cabale*) originally meant intrigue or an intriguing body, it has acquired a specially bad sense through mere accident.

Generalization and specialization are effected by Association aided by Imagination.

It may be mentioned in this connection that the generalization and specialization of terms are effected by the laws of association, supplemented at times by constructive imagination. The laws of association include similarity and contiguity; and constructive imagination involves either separation or combination of elements. (*a*) Thus, 'light' as dispelling darkness is likened to knowledge as removing ignorance; and hence light is used to signify knowledge itself. Similarly, 'fire' may stand for zeal or irascibility, and 'backdoor' for

Thus, the meanings of words are modified by (*a*) similarity,

* A distinction is sometimes drawn between the *first-intention* and the *second-intention* of a term. The first-intention is the common or popular use, which is generally vague and indefinite, while the second-intention is the special or scientific use, which is generally definite and precise. Thus, 'beasts' in the sense of oxen, 'birds' in the sense of partridges indicate the second-intention of the terms, as distinguished from their first-intention or general use.

any door meant for servants, even though it be in front of a house. (*b*) Suggestion by contiguity illustrates the figure known as Metonymy, as when we use 'sceptre' for royal authority, 'cradle' for infancy, 'grey hair' for old age, or 'grave' for death. (*c*) Separation of elements, found conjoined in experience, is illustrated in the figure called Synecdoche, as when Swift writes—

(*b*) contiguity,

(*c*) separation,

> "I do the most that friendship can,
> I hate the Viceroy, love the man."

We similarly use 'His Majesty' for king, 'His Excellency' for a governor or viceroy, 'His Holiness' for Pope. In all these cases we separate a quality or feature from an object, which is thus viewed in an ideal form by reference to it. (*d*) Combination of elements is illustrated in the union of words which separately indicate different things. Thus, 'coach-man,' 'red-hot,' 'far-sighted,' 'hailstones,' 'impossible,' 'unwise,' 'endless,' indicate composition —either of different words or of a prefix or suffix with a main word.

and (*d*) combination.

Contiguity sometimes effects great changes in the meanings of words.

The change effected by contiguity is sometimes so great that a word originally signifying one thing may subsequently come to imply quite a different thing through some links of association into which contiguity enters very prominently. The word 'rival,' for example, illustrates how a word originally applied to a thing by reason of one quality may gradually acquire the additional meaning of some associated quality, and thence be extended to things possessing the second quality alone. "*Rivals*, in the primary sense of the word," says Trench,

Examples.

"are those who dwell on the banks of the same stream. But since, as all experience shows, there is no such fruitful source of contention as a water-right, it would continually happen that these occupants of the opposite banks would be at strife with one another in regard of the periods during which they severally had a right to the use of the stream, turning it off into their own fields before the time, or leaving open the sluices beyond time, or in other ways interfering or being counted to interfere, with the rights of their opposite neighbours. And thus *rivals* came to be used of any who were on any grounds in more or less unfriendly competition with one another." *Cf*, 'Pandoor'. Similarly the word 'pectoral' (from Lat. *pectus* the breast) implies, as an adjective, 'of, or pertaining to the breast'; but, as a substantive, it indicates 'an armour, ornament, or dress worn on the breast,' or even 'a medicine for diseases of the chest.' The word 'alphabet' has likewise come to mean the letters of any language arranged in the customary order from signifying at the outset the first two Greek letters, *alpha* and *beta*. Cf. 'Sandwich.' Such a usage is called the **transitive application of words.** "Suppose", says Dugald Stewart, "that the letters A, B, C, D, E, denote a series of objects; that A possesses some one quality in common with B; B a quality in common with C; C a quality in common with D; D a quality in common with E; while at the same time, no quality can be found which belongs in common to any *three* objects in the series. Is it not conceivable, that the affinity be-

Transitive application of words implies extension in their application through contiguity.

tween A and B may produce a transference of the name of the first to the second; and that, in consequence of the other affinities which connect the remaining objects together, the same name may pass in succession from B to C; from C to D; and from D to E? In this manner, a common appellation will arise between A and E, although the two objects may, in their nature and properties, be so widely distant from each other, that no stretch of imagination can conceive how the thoughts were led from the former to the latter." (*Philosophical Essays*, p 217.) Thus, the word 'impertinent' originally signified irrelevant, not pertaining to the matter in hand, but, by gradual and easy transitions, it has come to mean intrusive, meddlesome, unmannerly, insolent. The word 'letter' has likewise undergone a series of transitions from its primary sense of alphabet to written communication, literature, and even any article carried by post. "The word *gentleman* originally meant simply a man born in a certain rank. From this it came by degrees to connote all such qualities or adventitious circumstances as were usually found to belong to persons of that rank. This consideration at once explains why in one of its vulgar acceptations it means any one who lives without labour, in another without manual labour, and in its more elevated signification it has in every age signified the conduct, character, habits, and outward appearance, in whomsoever found, which, according to the ideas of that age, belonged or were expected to belong to persons born and educated

Examples.

in a high social position." (Mill, *Logic*, II, p. 229)

Having considered the popular use of terms, let us now turn our attention to their scientific use. The aim of science, as we have seen, is to reduce to system what may otherwise seem to be an indefinite multiplicity. (*Vide* Chap. I, § 7.) And, as Whewell points out, "System and Nomenclature are each essential to the other. Without Nomenclature, the system is not permanently incorporated into the general body of knowledge and made an instrument of future progress. Without System the names cannot express general truths, and contain no reason why they should be employed in preference to any other names." (*Novum Organon Renovatum*, p. 288.) The popular names of the different qualities, features, or classes are too numerous and ill-defined to be of any practical value. Science tries to reduce the number by rendering the classes exact and connected and their names definite and significant. In Botany alone, the known species of plants were about 10,000 in the time of Linnæus; and they are nearly ten times that number now. By the *Binary Method of Nomenclature* (Names by Genus and Species) Linnæus reduced the number considerably. Thus, about 1,700 Generic Names, with a moderate number of Specific Names, were found by him sufficient to designate the different species of plants then known. His method has, therefore, been generally followed by scientists concerned with classification. The method consists in using single

The scientific use of terms is precise and based on system.

System and Nomenclature are interconnected.

Science economizes energy by reducing the number of terms.

The Binary Method or the Method of Double Naming consists in using a limited number of generic names and qualifying adjectives

for the species.

names for the higher classes (which are necessarily fewer in number) and in describing the numerous lower groups by the generic names and qualifying adjectives.* "The scientific name of every plant consists of two words, a substantive and an adjective. The substantive is the name of the *genus*, as Brown or Jones may be the name of a family. The adjective indicates the *species*, as John, Thomas, or William indicates the individual member of a family." (Oliver, *Text-Book of Indian Botany*, p. 125.) The generic name precedes. Thus, the different species of Fig are referred to the genus *Ficus* and are described by qualifying epithets as *Ficus religiosa* (the Peepul), *Ficus elastica* (the India-rubber tree), *Ficus benghalensis* (the Banyan). Similarly, in Chemistry, the compounds are named by reference to their elements. The compounds of the Metals, for example, are named thus :—(*a*) compounds with chlorine, bromine, and iodine are called the *chlorides*, *bromides*, and *iodides*; (*b*) compounds with oxygen and with oxygen and hydrogen, the *oxides* and *hydroxides*; (*c*) compounds with sulphur and with sulphur and hydrogen, the *sulphides* and *hydrosulphides*; (*d*) compounds with sulphuric and sulphurous acids, the *sulphates* and *sulphites*; (*e*) compounds with phosphoric acid, the *phosphates*; and (*f*) compounds with carbonic acid, the *carbonates*. We find likewise in Zoology the same method followed. The

Examples.

*One of the Linnæn maxims is that the generic name must be fixed before an attempt is made to form a specific name; "the latter without the former is," as he observes, "like the clapper without the bell." (Art. 219.)

genus *Felidæ*, for example, is divided into different species called *Felis leo* (lion), *Felis tigris* (tiger), *Felis leopardus* (leopard), *Felis concolor* (puma), *Felis lyncus* (European lynx), *Felis catus* (wild cat). This **method of double naming** (or the **binary method**, as it has sometimes been called) has the advantage of not only relieving the memory by considerably diminishing the number of independent names, but also of helping the understanding by indicating the relation in which a group or species stands to its appropriate higher class or genus. Sometimes the exact composition of a compound is conveniently indicated by the name itself : the compounds of chlorine and oxygen, for example, are represented as *chlorine monoxide* (Cl_2O), *chlorine trioxide* (Cl_2O_3), and *chlorine tetroxide* (Cl_2O_4). Scientific names, accordingly, tend to relieve the memory, enlighten the understanding, and render knowledge precise and systematic.

Scientific names relieve the memory, enlighten the understanding, and render knowledge accurate.

§ 5. Requisites of Scientific Language.

Scientific terms should be definite, precise, and systematic.

Scientific names should be as definite, precise, and systematic as possible. To avoid the unnecessary multiplication of terms, appropriate words, already current in a language, may be adopted in science with suitable qualifications. The conditions of accurate expression, essential to every science, may briefly be said to be the following :—

Conditions of the Scientific Use of Terms :

(1) *Every important meaning must have an appropriate name for it.* It implies that there should be (1) an adequate *nomenclature* for the different classes or groups of objects, (2) a comprehensive *terminology* for describing their various qualities or

(1) No important meaning should be without an appropriate name. There should,

therefore, be—

parts, and (3) appropriate *names for indicating the relations* of connected groups included in a classification.

(1) an adequate nomenclature;

(1) Nomenclature implies, as indicated in the last section, that there should be distinct names (generally short) for the different higher classes or orders; and that the names of the lower groups or species should generally be formed of these names and qualifying expressions. Thus, in Geology, there are names for classes of rocks and strata; in Mineralogy, names for the species and varieties of minerals; in Chemistry, names for the elements and their compounds; and in Botany and Zoology, names for the several species, genera, families, and orders.

(2) a comprehensive terminology with suitable names for

(a) the physical,

(b) metaphysical,

and (c) dynamic elements of things;

(2) Terminology implies that there should be suitable names for the (*a*) physical, (*b*) metaphysical, and (*c*) dynamical elements of things. (*a*) The names of the physical parts are the names of members or organs, such as nerve, muscle, head; stalk, petal, sepal; plinth, frieze, cornice. (*b*) The names of the metaphysical constituents are the names of qualities or features, with their various degrees and forms, such as size, figure, weight; hard, soft, elastic; red, yellow, green; sweet, bitter, sour; *etc.* (*c*) The names of the dynamic elements of objects are the names of processes and activities which enter into the conception of force or energy, such as causation, origination, decay, birth, growth, death, tendency, resistance, refraction, *etc.*

and (3) appropriate names of

(3) Appropriate names of the different groups related as members of a supreme class or kind are

also essential to indicate their mutual relations or connections. Thus, the terms 'kingdom,' 'class,' 'order,' 'genus,' 'co-ordinate species' point out the relations in which certain groups stand to others. "According to the laws of Botanical Nomenclature adopted by the International Botanical Congress, held at Paris in August 1867, no less than twenty-one names of classes are recognised—namely, Kingdom, Division, Sub-division, Class, Sub-class, Cohort, Sub-cohort, Order, Sub-order, Tribe, Sub-tribe, Genus, Sub-genus, Section, Sub-section, Species, Sub-species, Variety, Sub-variety, Variation, Sub-variation." (Jevons, *Principles of Science*, p. 727.)

relations.

(II) The other great condition of accuracy and precision is that *Every name must have a definite meaning attached to it.* The necessity of this rule is obvious. The use of terms in vague and ambiguous senses is a fruitful source of confusion and fallacy. Hence, in every science, the utmost care should be bestowed at the outset on clear and accurate definitions of the terms subsequently employed, so as to preclude the possibility of future error and misunderstanding. We find this procedure now generally adopted in many sciences, such as Geometry, Dynamics, Physics, and Chemistry.

(II) No name should be without a definite meaning.

To avoid the unnecessary multiplication of words, popular names may be adopted in science when they are found to suit our purpose. And, as popular names are divisible into the same classes into which scientific terms are divisible—*viz.*, names of things, classes, parts, qualities, and activities—

When suitable popular names are available, they may be adopted in science;

popular names may often be of service in science. As, however, ordinary language is generally loose and vague, we should definitely settle the meanings of popular terms when they are adopted in a science. We may do so either (*a*) by reference to their etymological or central signification or (*b*) by qualifying epithets and interpretative clauses. When, however, the scientific meaning cannot thus be grafted on a common term without violence to language, it is better to coin new words for special purposes than to use current words in unheard-of-senses, which lead only to confusion and obscurity.

But their meanings should be rendered exact by qualifying epithets and explanations.

But settled meanings should not be rudely disturbed.

"The precautions to be observed in re-adjusting the signification of terms," says Bain, "are these:— First, important meanings in current use, or meanings at the base of important predications, should not be disturbed; secondly, the associations of powerful sentiment should not be reversed." (*Logic*, II, p. 175.)

Popular language may be improved when we (1) fix the meanings of terms by reference to their derivation, (2) resist the tendencies towards generalization and specialization, and (3) divert the use of superfluous terms to allied meanings

It may be mentioned in this connection that we may also try to improve popular language by rendering the meanings and uses of current terms precise and definite. We may do so in three principal ways:—(1) The meanings of terms should, as a rule, be settled by reference to their derivation. (2) We should resist the tendency towards excessive generalization or specialization: we should not make terms unduly wide or narrow by unwarrantably using them in an indefinite or restricted sense. (3) When two or more terms are exactly synonymous, it would be conducive to precision and perspicuity to divert some of them

to neighbouring or allied meanings which have no appropriate expression.

having no appropriate names.

§ 6. **Exercises.**

1. Point out the importance of Language in general and of Scientific Names in particular.

2. Distinguish between intuitive and symbolical knowledge, indicating their relative importance.

3. Determine the relation of Definition, Classification, and Naming. Is a Term or Name an elementary or a complex logical product?

4. Distinguish between Scientific Terminology and Nomenclature, and indicate their relative uses.

5. 'The popular use of terms is generally vague and uncertain.' Why? How can the defects be remedied?

6. What do you understand by the Generalization and Specialization of Terms? How are they caused? Give illustrations.

7. Explain and illustrate what is meant by the Transitive Application of Words.

8. What do you understand by Double Naming? Point out its scientific importance.

9. What are the requisites of Scientific Knowledge? Is it advisable to use popular names for scientific purposes? If so, when and to what extent?

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BOOK V.

METHOD.

CHAPTER XXVIII. omitted whole

EXAMINATION OF MATERIALS.

As rational beings, we employ the different logical processes for the attainment of a definite end.

§ 1. Importance of Method. Having examined the several forms of the logical processes let us now turn our attention to the way in which they may be best adapted to some end—either theoretical or practical. Being rational creatures, we generally employ definition or division, classification or naming, deduction or induction not as isolated operations but as different steps towards the attainment of an object, which may be the acquisition or communication of knowledge, the gratification of feeling, or the control over some circumstance or agency. To be accurate, we define ; to be systematic, we classify ; and to widen our knowledge, we have recourse to observation and inference. As, however, accuracy, system, and increase of knowledge are all interconnected, we are disposed to employ all these processes in regular succession to arrive at the desired result. And it is on this systematic employment of the different steps that the success or failure of an undertaking generally depends. Thus, the mere isolated correctness of the logical processes explained in the preceding Books is not of much

Mere isolated correctness of these processes is not of much

consequence; to be of real value, they must converge on a definite purpose and contribute to its realization. Hence, Method or systematic procedure may be described as the crown of all logical operations, which, without it, can at most have a momentary or fragmentary value. And the difference between good and bad intelligence often lies more in the proper or improper use of Method than in the bare detached uses of the several logical operations. Descartes very well observes, "The power of judging aright and of distinguishing Truth from Error, which is properly what is called Good Sense or Reason, is by nature equal in all men; and the diversity of our opinions, consequently, does not arise from some being endowed with a larger share of Reason than others, but solely from this, that we conduct our thoughts along different ways and do not fix our attention on the same objects. For to be possessed of a vigorous mind is not enough; the prime requisite is rightly to apply it. The greatest minds, as they are capable of the highest excellencies, are open likewise to the greatest aberrations; and those who travel very slowly may yet make far greater progress, provided they keep always to the straight road, than those who, while they run, forsake it." (*Discourse on Method*, Veitch's translation, pp. 3—4.)

importance without their systematic arrangement.

Thus, Method may be viewed as the crown of logical operations.

The differences observable among men are due more to the proper or improper use of Method than to natural endowments.

§ 2. Preliminary Conditions of Method. Method, as Kant says, is "procedure *according to principles.*" (*Critique of Pure Reason*, Meiklejohn's translation, p. 516.) We keep steadily before our

Method is 'procedure according to principles.'

mind some end to be attained and regulate our steps by such principles as are conducive to its realization. We have thus to observe with care certain facts and circumstances and to find out the laws which govern them, in order that we may systematically use the appropriate means for arriving at the desired result. "Although in every question," observe the authors of *The Port-Royal Logic*, "there is something unknown, otherwise there would be nothing to seek, it is, nevertheless, necessary that even that which is unknown should be marked out and designated by certain conditions which may determine us to seek one thing rather than another, and which may enable us to judge, when we have found it, that it is the thing of which we were in search." (Baynes' translation, pp. 310-311.) As the ultimate end of knowledge is the correspondence of ideas with facts, every sound method must estimate these aright that it may not terminate in fiction; in order that a superstructure may be stable, its foundation must be secure. Whether we proceed from facts to principles or from principles to facts, a correct estimate of facts is always essential to the validity of the result. But, though careful observation plays a prominent part in all sound method, yet other conditions are also necessary to render it efficacious. The chief preliminary conditions of sound method may briefly be indicated thus:—

It involves a careful examination of facts.

The preliminary conditions of sound method are:—

(1) Correct observation,

(1) *Correct Observation.* We should carefully and accurately study the facts connected with the

inquiry in hand. In such study we should be guided by the following considerations :—

which requires

(a) direct apprehension or testimony,

(a) Direct apprehension or testimony should always be preferred to indirect. The great danger of observation is the confusion of perception with inference : in many cases we mistake a conclusion for a percept. We imagine, for example, that we directly know by sight the distance, solidity, or weight of an object, when really it is inferred from certain signs. And we have already seen that the value of testimony diminishes as it passes from one person to another. (*Vide* Chap. XXI, § 9.) Once D said to E (so the story goes) that X had vomited three black crows. E asked D whether he had seen it. D replied, he had learnt it from C, who in his turn said that he had not seen it himself but had learnt from B that X had vomited two (and not three) black crows : and in this way the report was finally traced to an eye-witness, who simply said that X had vomited something black—as black as a crow. Such is often the value of indirect testimony ; and hence it is generally inadmissible in courts of law. We should, therefore, always try to derive our knowledge from direct observation or testimony, whenever possible.*

(b) absence of bias,

(b) We should study with an unprejudiced mind, for bias often colours an object observed. We are often disposed to construe objects accord-

* When we are constrained to depend on indirect testimony (as in the case of historical or biographical incidents or astronomical observation), we should determine its value by reference to the authenticity of the record, the character of the reporter, and the consistency of the report with other facts proved independently to be true.

ing to our views. Men think that their experiences determine their ideas; but their ideas have often a reflex effect on their experience. Thus, friends are led by affection to under-estimate, while opponents are led by ill-will to over-estimate the fault of a person. Faddists are similarly disposed to construe facts in the light of their fads.

and (*c*) examination of relevant facts alone.

(*c*) Observation should be duly regulated, so as to include within its compass only the relevant facts, or facts bearing on the subject in hand, and not to cover indiscriminately any and all facts. In fact, the difference between observation and perception is that, while the one is regulated, the other is casual: observation is but well-regulated perception. As observation is thus under the control or guidance of a leading idea or principle, the facts observed must always be determined by it. Thus, while the zoologist may bring apes and men within the compass of his study, by reason of their similarity in respect of organic structure, the moralist is led to exclude the former from his province as devoid of any moral significance. The relevancy or irrelevancy of a topic in every case must, therefore, be determined by our end in view and the sphere of our inquiry.

Observation assumes two forms:

It may be mentioned in this connection that observation may assume either of two forms, according as its objects constitute the facts of the internal world of Mind or of the external world of Nature and Society.

Introspection and Sense-perception.

Introspection in the one case and Sense-perception in the other are the means employed to gather facts by direct observation. And, if Introspection prevails

in the mental and moral sciences, Sense-perception preponderates in the sciences of Nature.

(2) *Analysis and Synthesis.* Mere observation of facts, presented to the mind, is not adequate, however, for the purposes of a science. Most of the facts known in adult life are of a complex character. Adequately to explain them, therefore, we must break them up into their constituent elements and discover the laws of their combination. It is thus necessary to have recourse to the two methods of Analysis and Synthesis. Analysis implies the separation of elements which are found together in a concrete object or experience, while Synthesis means the combination of elements for the reconstruction of such a product. Synthesis is not "the unabstracted concrete" : it is "combining *after* analysing ; it is using the results of analysis with a view to construction." (Bain, *Logic*, Part II, p. 397.)

(2) Analysis and Synthesis.

Analysis implies separation of elements ; while synthesis, their combination.

Analysis and synthesis assume two distinct forms owing to a difference in the materials on which they are employed. *Physical* or *chemical analysis* implies the actual separation of elements which go to constitute a compound, while *logical* or *psychological analysis* implies the ideal separation of elements entering into a complex fact or notion. Similarly, physical or chemical synthesis is the actual composition of elements for the production of a desired compound, while logical or psychological synthesis is the ideal reconstruction of a complex notion out of the elements discovered by prior analysis. Synthesis in either case supple-

While physical analysis is actual separation, logical analysis is ideal separation.

Similarly, physical synthesis is actual composition, while logical synthesis is

ideal reconstruction.

Synthesis proves the correctness and adequacy of prior analysis.

ments analysis to verify its results: the correctness and adequacy of analysis are proved by the subsequent synthetic reconstruction of the complex fact out of the elements and according to the laws already discovered by analysis.

(3) Definition and Classification of facts.

(3) *Definition and Classification.* As the result of a careful estimate of facts and their due analysis, we must correctly define and classify them, to avoid future mistake or confusion. We have already indicated in the preceding Book, the importance of these processes in every scientific investigation.

(4) Employment of the rules of Inductive and Deductive Inference.

(4) *Use of Inductive and Deductive Inference.* To understand things aright, it is not sufficient that we should merely observe, analyse, classify, and define them; it is further necessary that we should discover the laws which govern them. To understand a fact is to assimilate it and to discover causal links which bring all similar facts together. (*Vide* Chap. XXIV, § 2.) Thus, the principles of induction and deduction must be employed on appropriate materials to arrive at correct generalizations and deductions from them. All the experimental methods, with the connected processes of elimination, hypothesis, and verification should, therefore, be used to elucidate physical and psychological facts.

(5) Thoroughness.

(5) *Comprehensiveness.* Our treatment of a subject should be comprehensive and full and not incomplete and imperfect. Every science must take an exhaustive survey of *all* the facts coming within its province or their representative instances and try to explain them, not merely in isolation, but in their mutual connection and bearing. As

Descartes says, "We should divide each of the difficulties under examination into as many parts as possible and as might be necessary for its adequate solution; and in every case we should make examinations so complete, and reviews so general that we might be assured that nothing was omitted." (*Discourse on Method.*)

The above conditions indicate merely the steps we should take to ensure the correctness and adequacy of the materials which constitute the subject-matter of an inquiry. How to arrange these materials, either for discovery or for instruction, falls within Method proper, which we shall consider in the next chapter.

§ 3. Knowledge Fit for Methodical Treatment. Knowledge is fit for methodical treatment when it is clear, distinct, and exact If our ideas be vague and inaccurate, then they generally overlap, thereby preventing system and begetting confusion. Methodical treatment is aided when the following marks are found in knowledge :—

Knowledge, to be fit for methodical treatment, must be

(1) *Clearness.* Knowledge is said to be clear when its object as a whole can be distinguished from other wholes. Thus, we may be said to have a clear knowledge of the horse when we can distinguish it from other animals, such as the ass, cow, and buffalo. Absence of clearness means obscurity and vagueness.

(1) clear,

(2) *Distinctness.* Knowledge is called distinct when the several parts or qualities of its object are definitely known. Thus, a good painter or sculp-

(2) distinct,

tor may be said to have not merely a clear but also a distinct knowledge of the objects with which he deals. Common knowledge, for example, of the horse may be clear, though not distinct. Indistinctness, no doubt, often leads to obscurity as well.

and (3) accurate,

(3) *Accuracy.* Knowledge is accurate when it exactly corresponds to facts. Clearness and distinctness generally secure accuracy; but we may have vivid ideas of the parts as well as of the whole, though they may not quite agree with facts. We may have a rough idea of the whole and an inadequate idea of the parts; and thus our knowledge may not be accurate. To be accurate, it must be adequate and consistent. When our knowledge exceeds its proper limits, then also it becomes inaccurate, as when foreign elements are incorporated into an idea. It may be mentioned in this connection that the accuracy of knowledge is preserved only so long as symbolical knowledge can be converted into intuitive. When such a possibility ceases, knowledge is again likely to become inaccurate and vague. (*Vide* Chap. I, § 3 and Chap. XXVII, § 1.)

Accuracy involves adequacy and consistency.

Symbolical knowledge, when not convertible into intuitive, becomes a source of error.

§ **4. Exercises.**

1. Point out the importance of Method in ordinary and scientific inquiries.

2. What are the preliminary conditions of sound Method? Are they essential to every inquiry?

3. Distinguish between Perception and Observation. Indicate the forms and conditions of correct Observation.

4. Distinguish between Physical and Logical Analysis. How is Synthesis related to Analysis?

5. Determine the features which render knowledge fit for methodical treatment.

CHAPTER XXIX.

Disposition of Parts.

§ 1. **Definition of Method.** Method proper consists in so adjusting the parts of an inquiry or discourse as to render the whole easily intelligible. Method, accordingly, implies a multiplicity of parts systematically arranged for a definite purpose, which may be either the acquisition or the communication of knowledge. It involves reference to an end (which is either discovery or instruction) and a guiding sense of what is needed to promote that end. This guiding sense is determined by previous experience, indicating approximately the way in which we should proceed as well as the scope of investigation or treatment in any case. Having, thus, before our mind the end to be achieved and the sphere of inquiry, we have to adapt the different parts to one another in such a manner as to render our treatment at once clear and comprehensive. Method has, accordingly, been defined by the Port-Royalists as "The art of disposing well a series of many thoughts, either for discovering truth when we are ignorant of it, or for proving it to others when it is already known." (Baynes' translation, pp. 308—309.) The essence of Method lies in so arranging our reasonings on any subject as to produce their combined effect on the mind eager for knowledge.

Method is the due arrangement of the parts of an inquiry or discourse so as to render it clear.

Method involves (1) reference to an end and (2) a guiding sense of fitness.

Definition of Method by the authors of Port-Royal Logic.

The essence of Method lies in its cumulative effect.

§ 2. Natural Order of Arrangement.

The natural order of arrangement is either physical, psychological, or logical.

As the peculiarity of Method is the proper arrangement of parts for the production of a desired effect, the chief question connected with its use is, How to arrange the parts in order to attain the end? It is generally said that we should treat of things in their "natural order" to make them easily intelligible. But the natural order may be (1) physical, (2) psychological, or (3) logical order. (1) We may, for example, explain the character of crystals by indicating the mode of their formation; or (2) we may commence our inquiry with the crystals as formed and then try to discover by analysis and abstraction the general conditions which determine their structure. And (3) in either case we are to arrange our thoughts in such a way that the comparatively simple and independent may precede what is relatively complex and dependent. On a closer examination we find that the first or physical order is possible only when we have a prior knowledge of the elements and laws which combine to produce the phenomenon under investigation. This may be called the synthetical method, proceeding from principles to facts. The second or psychological order is the method of analysis in which we proceed from facts to principles. In one, we try to trace the way in which Nature works from elements to a complex product; while in the other, we try to discover the mode of her operation by starting with the product as supplied to the mind. Analysis and

All these

Synthesis, then, are the two principal methods employed to explain the constitution and genesis of things. The third or logical order also resolves itself into either of these two methods, according as our reasonings proceed from facts to principles or from principles to facts, *i.e.*, as they are essentially inductive or deductive. The two fundamental methods, therefore, which regulate every inquiry and exposition are Analysis and Synthesis. Let us consider them a little fully in a separate section.

procedures are finally resolvable into two methods, Analysis and Synthesis.

§ **3. Analytical and Synthetical Procedure.** The general principle of method to be observed in all inquiry is that we should proceed from the simple to the complex, from the familiar to the obscure. As, however, simplicity or familiarity varies with individuals, the methods employed by them necessarily vary. To a child or beginner, the individual is more familiar than the general, the concrete more familiar than the abstract. And, in a certain sense, the individual or concrete is also simple to him—simple psychologically, though not logically. In the history of mental development we find that concrete things are known earlier than their abstract qualities or relations, individuals before the classes to which they belong. Thus, generally, the method of discovery is the method of analysis. But, with the increase of knowledge, we become familiar with the elements and laws which enter into the composition of concrete and individual things. We thus learn to retrace our

The general rule of method is to proceed from the simple to the complex, from the familiar to the unfamiliar.

Simplicity and familiarity, however, are relative to individuals.

Generally, Analysis is the method of discovery; and Synthesis, the method of exposition.

steps backwards, from the elements and laws to their complex products. This is the synthetical method. To discover, therefore, the way in which Nature works, we must previously pass through the way which reveals her procedure. The synthetical method, therefore, properly presupposes the analytical. But, situated as we are in society, we are often saved the trouble of personal investigation owing to the instruction we receive from others. Hence, a learner may acquire knowledge synthetically without prior analysis. He has then only to accept the general principles enunciated by his instructor and to follow his exposition synthetically. Thus, though primarily synthesis presupposes analysis, yet, situated as we are, we can often pursue the two methods independently of each other.

The Synthetical method ordinarily presupposes the Analytical, though as social beings, we may pursue the two methods independently.

Uses of the two Methods :

The following points in this connection deserve special notice :—

(1) Analysis regulates induction ; while synthesis, deduction.

(1) The Analytical Method regulates, as we have seen, our inductive investigations ; while the Synthetical Method, our deductive inferences. In the one case we start with concrete cases and then try to discover the conditions and principles by analysis ; while in the other, we synthetically connect principles with cases to see the conclusion justified by both of them.

(2) Analysis favours discovery and synthesis, exposition.

(2) The Analytical Method, as explained above, is generally the Method of Discovery, while the Synthetical Method is the Method of Exposition. Thus, we come to know principles and characteristics governing classes, by Inductive Inferences,

Definitions, and Classifications, which involve the Analytical Method, while we explain and illustrate such principles and characteristics by Deductive Inferences, Definitions, and Divisions, involving the Synthetical Method.

(3) The Analytical Method is suitable for children, who are familiar with concrete things; while the Synthetical Method is adapted to the requirements of adults, who are more or less familiar with general truths. The object-lessons of infancy, which constitute so prominent a part in the kindergarten system of education, involve an appeal to observation, abstraction, and analysis, which more effectively secure accuracy, distinctness, and clearness of knowledge than can possibly be attained by a mere synthetic exposition from principles. It should, however, be borne in mind here that the analytic method employed for instruction is not exactly the same as is employed for original research. Though, in both the cases, the procedure is from facts to principles, yet in the case of instruction, the course is more straight, regular, and easy than in the other case. In original investigations, the materials are not so systematically arranged, nor is the analytic procedure so direct and short as in exposition, in which only the appropriate materials and lines are chosen by the instructor as furnished by his previous knowledge.

(3) Analysis suits children better than adults.

Difference between the analytic method as employed for instruction and that as employed for discovery.

(4) The analytical and synthetical methods, though distinct, are at times employed together to establish a position beyond dispute: one of these methods is then used to verify the results

(4) The full force of logical method is illustrated when both

the methods are employed together.

arrived at by the other. As the best conclusive evidence is reached when a point is proved both deductively and inductively, so the full force of logical method is illustrated when analysis and synthesis supplement each other.

The general conditions of methodic procedure or treatment.

§ 4. **The Rules of Method.** The general conditions of all methods are that we should proceed systematically and by easy transitions from the simple to the complex and that the connection between any two steps should, as a rule, be furnished by causal connection or logical sequence. In the absence of causal or logical succession, we should have recourse to the psychological order as determined by the laws of association. When steps are arranged in this way, there will be an easy flow of ideas, readily grasped and combined in a system. But, besides these general conditions, we may mention some special rules applicable to the two Methods separately.

The special rules of the Analytical Method.

(I) *Rules for the Analytical Method.*

(1) We should observe carefully the facts to be explained with an unprejudiced mind.

(2) We should distinguish the facts from other analogous facts and thus demarcate the exact sphere of our inquiry.

(3) We should exhaustively consider the different parts, elements, or aspects of the facts to be explained.

(4) We should ascertain the laws governing these parts, elements, or aspects as well as the laws governing their relations.

(5) We should try to trace these laws to higher laws, so that the laws governing the features, elements, and relations may be considered as derivative and certain. (*Vide* Chap. XXIII, § 3 and § 4.)

(6) We should consider the facts and their parts or elements in their natural order proceeding from the less to the more general.

(7) We should take a comprehensive survey of the sphere of our inquiry in order to be sure that nothing is left out which is calculated to throw light on it.

The specia rules of the Synthetical Method.

(II) *Rules for the Synthetical Method.* These rules are thus laid down by the authors of *The Port-Royal Logic* :—

"*Two Rules touching Definitions.*

1. Not to leave any terms at all obscure or equivocal, without defining them.

2. To employ in definitions only terms perfectly well known, or already explained.

"*Two Rules for Axioms.*

3. To demand as axioms only things perfectly evident.

4. To receive as evident that which requires only a slight attention to the recognition of its truth.

"*Two Rules for Demonstrations.*

5. To prove all propositions which are at all obscure by employing in their proof only the definitions which have preceded, or axioms which have been granted, or propositions which have been already demonstrated.

6. Always to avoid the equivocation of terms by substituting mentally the definitions which restrict and explain their meaning.

"*Two Rules for Method.*

7. To treat of things, as far as possible, in their natural order, by commencing with the most general and simple, and explaining everything which belongs to the nature of the genus before passing to its particular species.

8. To divide, as far as possible, every genus into all its species, every whole into all its parts, and every difficulty into all its cases." (Baynes' translation, pp. 346—347.)

§ 5 **Exercises.**

1. Define Method and explain its precise character and scope.

2. What is the natural order of inquiry or exposition? Is it the same always?

3. What are the general rules of Method? How does the Method of Discovery differ from that of Exposition?

4. State and explain the rules of the Analytical and the Synthetical Method respectively.

5. Point out the proper uses of the Analytical and the Synthetical Method. Is there any difference between the Analytic Method as employed for instruction and that employed for discovery?

6. Indicate any difference in the uses of Inductive and Deductive Definitions and Classifications.